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Simultaneous graphic generalisation

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Aim of the project

Determine the applicability of simultaneous graphic generalisation for:

* map production generalisation
* real-time generalisation.

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Disposition

- Simultaneous graphic generalisation
- Setting weights of the constraints
- Case studies
- Conclusions

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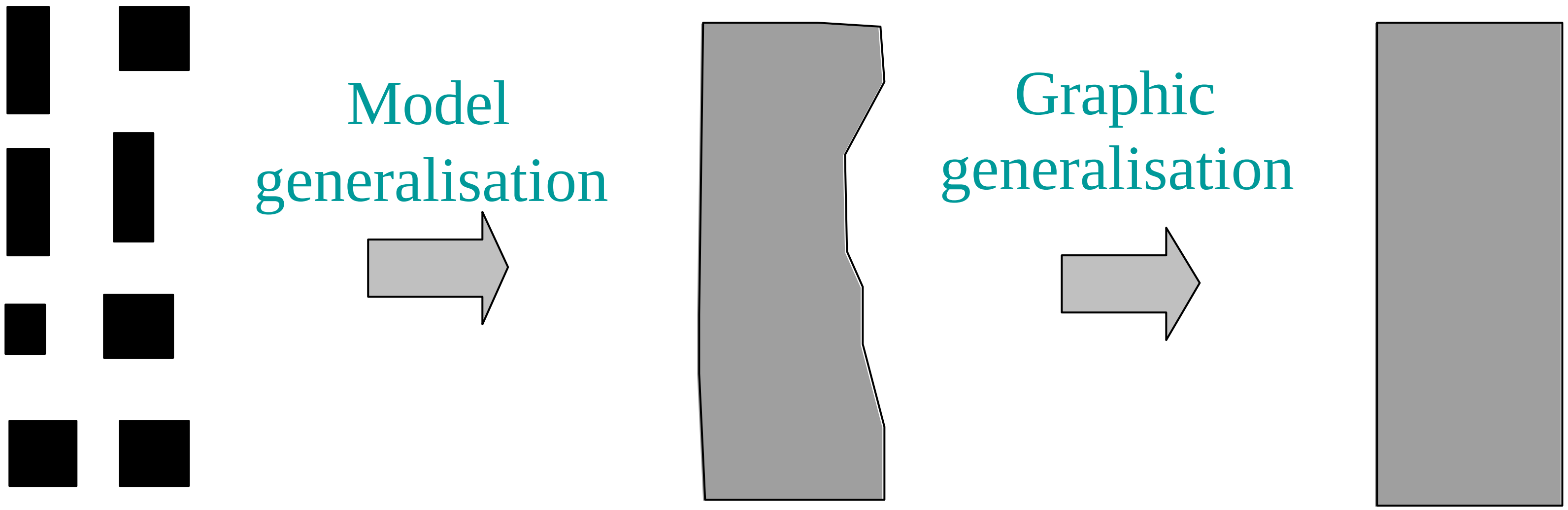
Different types of generalisation



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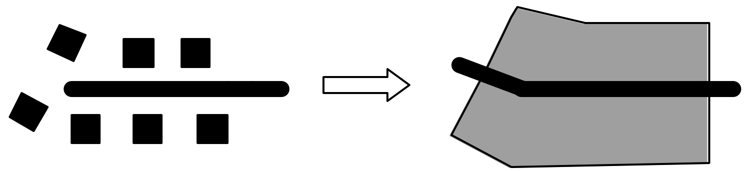
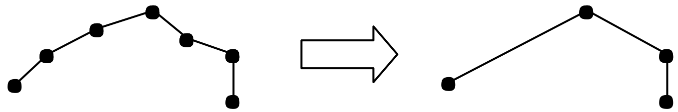
Generalisation Operators

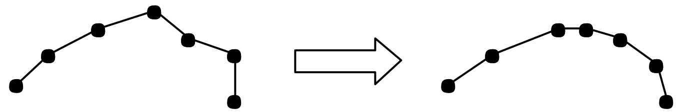
Aggregation



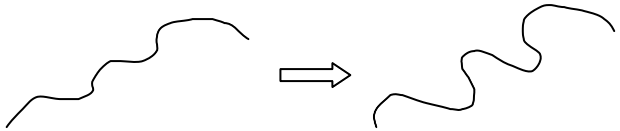
Simplification



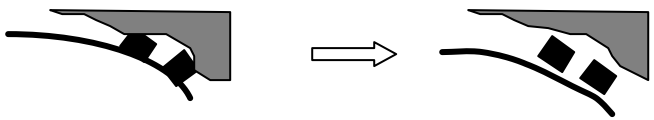
Smoothing



Exaggeration



Displacement



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Sequential Approach

Model generalisation

- Selection ↓
- Aggregation ↓
- Collapse

↓

Fine tuning

- Simplification ↓
- Exaggeration ↓
- Smoothing ↓
- Displacement

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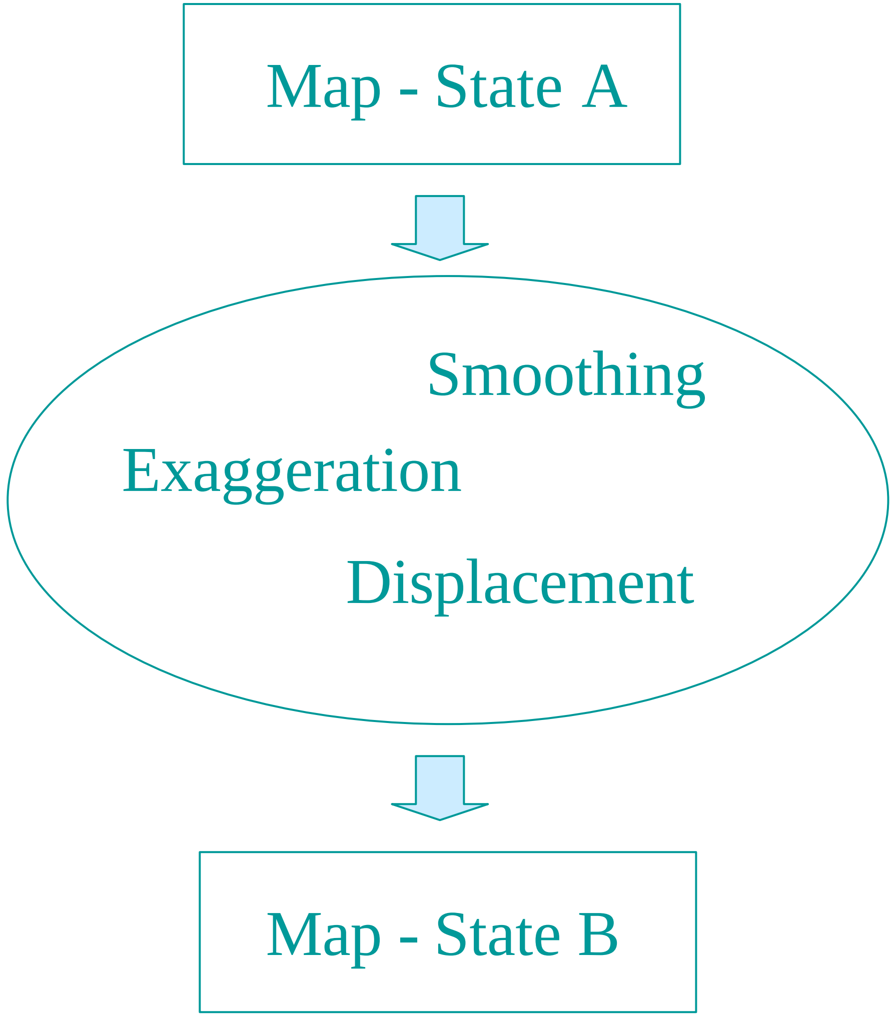
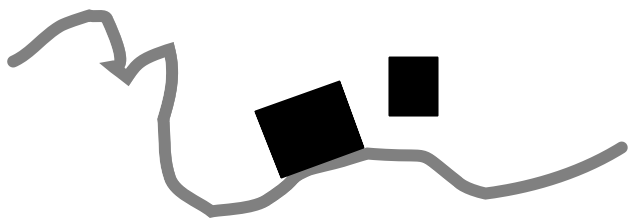
Simultaneous Approach - Graphic generalisation

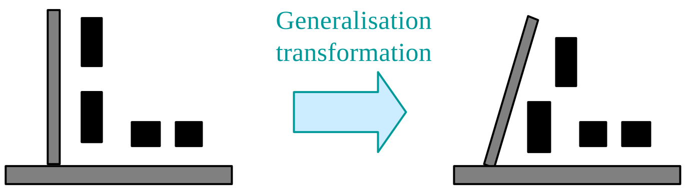


Constraints



Legibility constraints:

Simplification, Smoothing, Displacement, Exaggeration



Characteristic constraints:

Curvature, Segment length, Stiffness, Crossing

Position constraints:

Movement, Movement direction, Relative position

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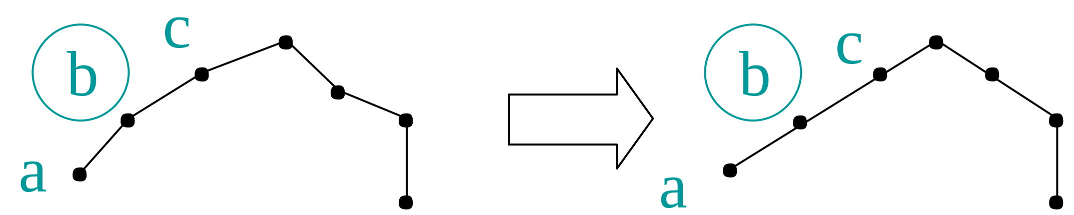
Simplification

- remove unnecessary points



$$-(1-d)\cdot\Delta x_a+\Delta x_b-d\cdot\Delta x_c=(1-d)\cdot x_a-x_b+d\cdot x_c$$

$$-(1-d)\cdot\Delta y_a+\Delta y_b-d\cdot\Delta y_c=(1-d)\cdot y_a-y_b+d\cdot y_c$$

where

$$d=\frac{d(a,b)}{d(a,b)+d(b,c)}$$

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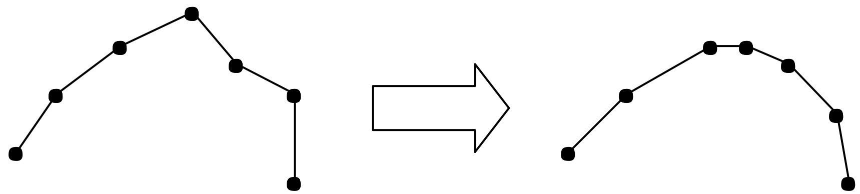
Smoothing
- make lines less angular

$$new\_x(i) = \frac{\sum_{u=-b}^{b} x(s_i + u \cdot distance) \cdot e^{-a \cdot (u \cdot distance)^2}}{\left\| \sum_{u=-b}^{b} e^{-a \cdot (u \cdot distance)^2} \right\|}$$

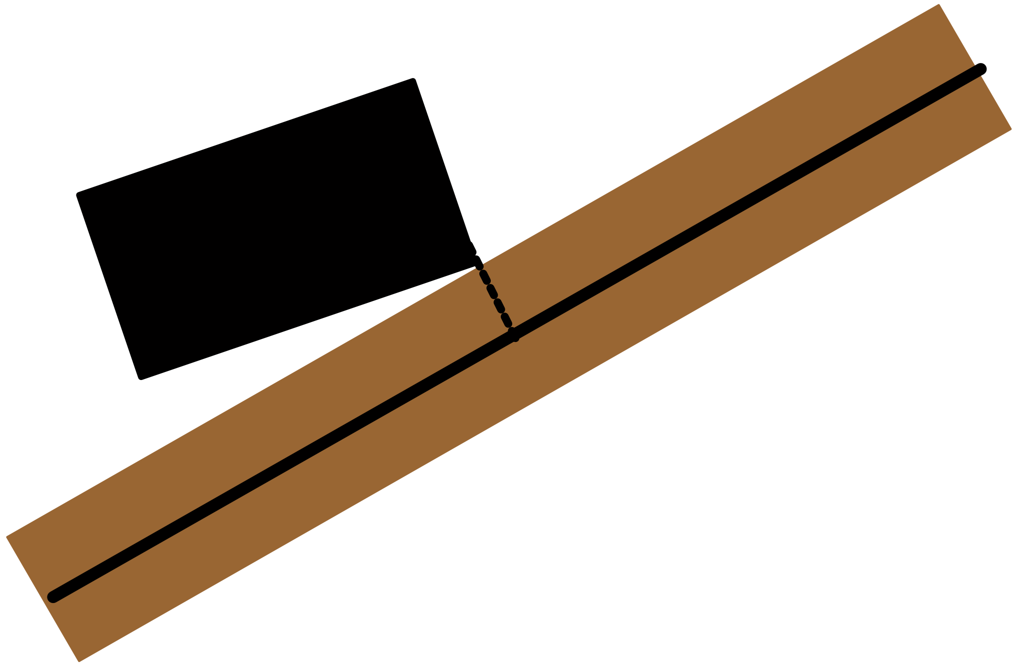


$$\Delta x_i = new\_x_i - x_i$$

$$\Delta y_i = new\_y_i - y_i$$

Displacement -

objects should not be in spatial conflict



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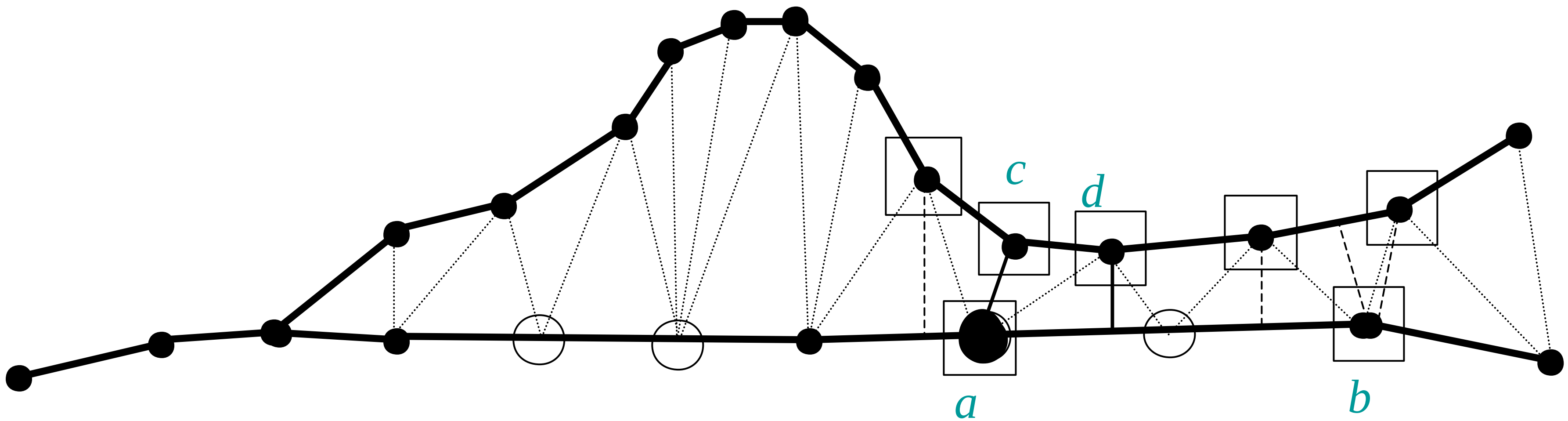
Rules for setting up displacement constraints



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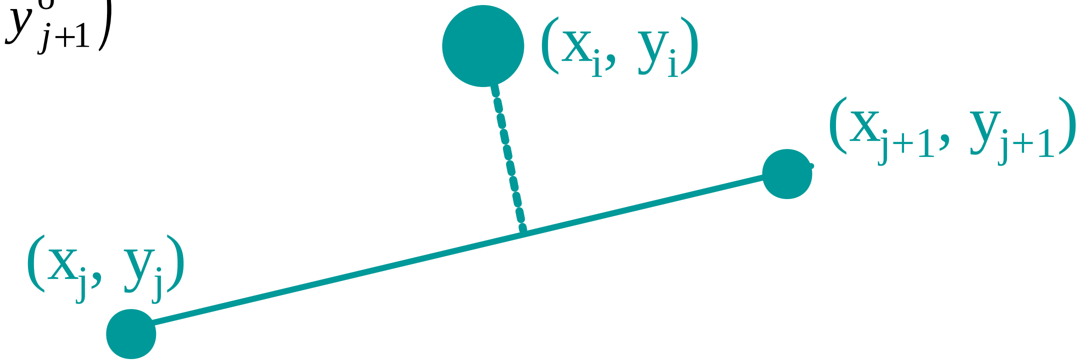
Spatial Conflict Constraints

$$line = \left\{ [x_l, y_l] \;\middle|\; a \cdot x_l + b \cdot y_l + c = 0 \right\},\ (x_l, y_l) \in (x_j < x_l < x_{j+1},\ \ y_j < y_l < y_{j+1})$$

$$distance\left(\left[x_i^{o}, y_i^{o}\right], line \right) = \left| \ a \cdot x_i^{o} + b \cdot y_i^{o} + c \ \right| / \sqrt{a^2 + b^2} =$$

$$= \mathrm{h}_k\left(x_i^{o}, y_i^{o}, x_j^{o}, y_j^{o}, x_{j+1}^{o}, y_{j+1}^{o}\right)$$



$$\left.\frac{\partial h_k}{\partial x_i}\right|_{\bar{\mathbf{x}}} \cdot \Delta x_i + \left.\frac{\partial h_k}{\partial y_i}\right|_{\bar{\mathbf{x}}} \cdot \Delta y_i + \left.\frac{\partial h_k}{\partial x_j}\right|_{\bar{\mathbf{x}}} \cdot \Delta x_j + \left.\frac{\partial h_k}{\partial y_j}\right|_{\bar{\mathbf{x}}} \cdot \Delta y_j +$$

$$+ \left.\frac{\partial h_k}{\partial x_{j+1}}\right|_{\bar{\mathbf{x}}} \cdot \Delta x_{j+1} + \left.\frac{\partial h_k}{\partial y_{j+1}}\right|_{\bar{\mathbf{x}}} \cdot \Delta y_{j+1} = distance\_value$$

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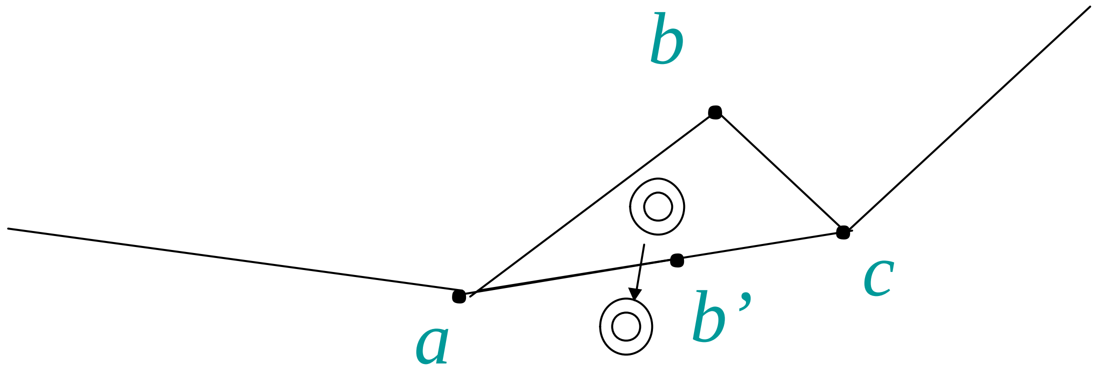
Spatial relationships betwen objects

Two desired properties

1) Simplify the line (remove point *b*)

2) The ring object is moved so that no spatial conflict occurs



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Least Squares Adjustment

The linear (overdetermined) equation system

$$\mathbf{Ax} = \mathbf{l} + \mathbf{v}$$

has the least square solution

$$(\mathbf{A}^t\mathbf{PA})\mathbf{x} = \mathbf{A}^t\mathbf{Pl}$$

where **x** contains the unknown point movements and **P** the weights

Conjugate Gradient Method

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Residual vector: $\mathbf{r}_i = \mathbf{u}_i - \mathbf{N}\mathbf{x}_i$

where $\mathbf{N} = \mathbf{A}^t\mathbf{P}\mathbf{A}, \quad \mathbf{u} = \mathbf{A}^t\mathbf{P}\mathbf{l}$

CGM proceeds by following iteration with the starting values

$$\mathbf{x}_o = \mathbf{0}$$
$$\mathbf{p}_o = \mathbf{r}_o$$

$$a = \frac{\mathbf{r}_i^t \mathbf{r}_i}{\mathbf{p}_i^t \mathbf{N}\mathbf{p}_i}$$

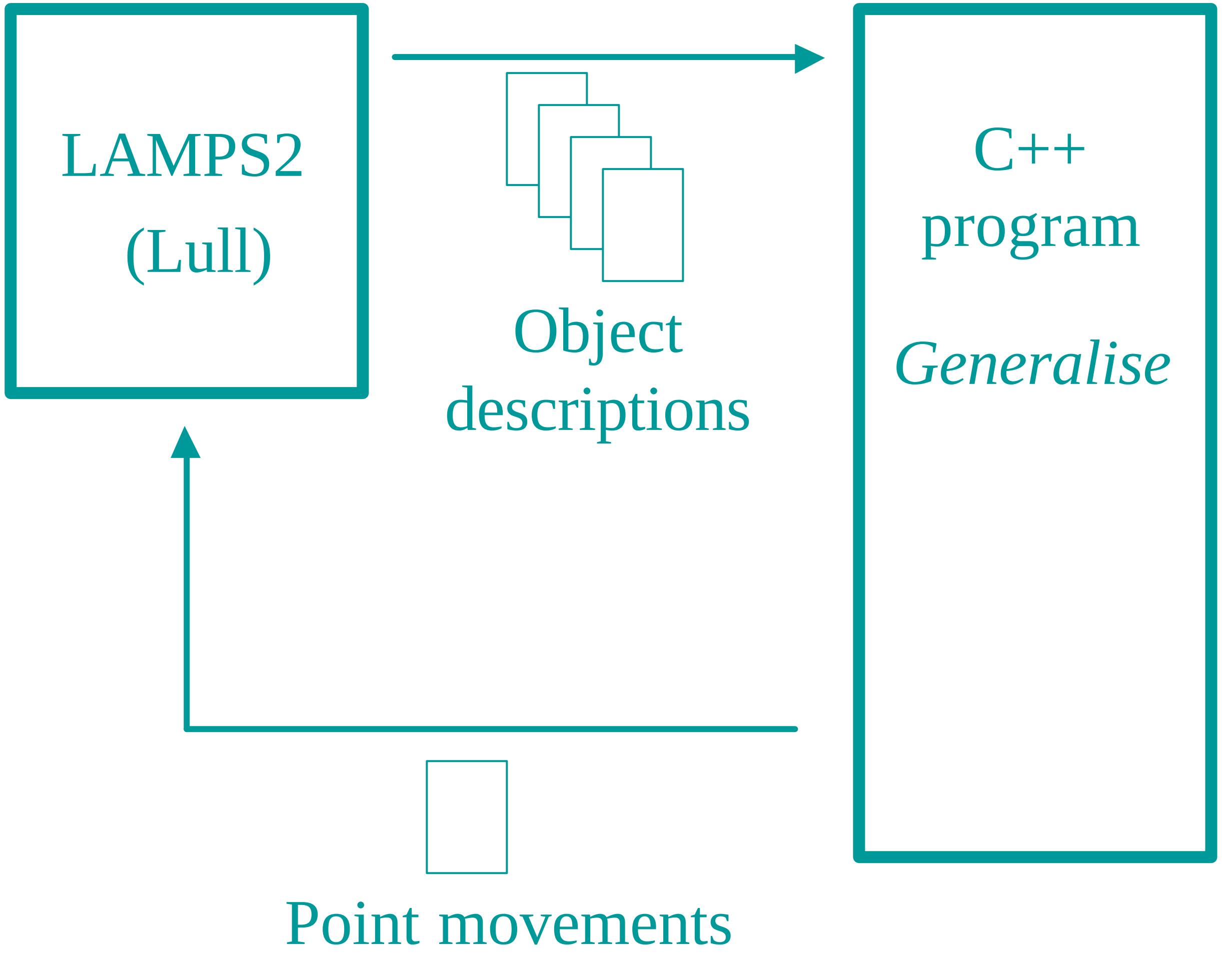
$$\mathbf{x}_{i+1} = \mathbf{x}_i + a\,\mathbf{p}_i$$

$$\mathbf{r}_{i+1} = \mathbf{r}_i - a\,\mathbf{N}\mathbf{p}_i$$

$$b = \frac{\mathbf{r}_{i+1}^t \mathbf{r}_{i+1}}{\mathbf{r}_i^t \mathbf{r}_i}$$

$$\mathbf{p}_{i+1} = \mathbf{r}_{i+1} + b\,\mathbf{p}_i$$

Implementation



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Program Generalise

Creating constraint Delauney triangles

Setting up constraints

Solving equation system

→ Triangulate

Jonathan Richard Schewchuck
USA

→ Conjugate gradient method routines

Tapani Sarjakoski
FGI, Finland

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Weight-setting strategies

- *Empiricism:* trial-and-error strategy

- *Machine learning:* use training set to find relationship between object properties and the weights

- *Constraint violation:* based on a priori estimates on the allowed violations

- *Variance component estimation:* posteriori statistical strategy that minimise total variance

Constraint violation strategy

The weights are set according to:

$$\mathbf{P}_{i,i} = \frac{1}{\left(allowed\ \ violation_i\right)^2}$$

Recommended strategy under the assumptions:

* The weights are functions of the type of constraints and type of objects only
* All the constraints are considered to be independent

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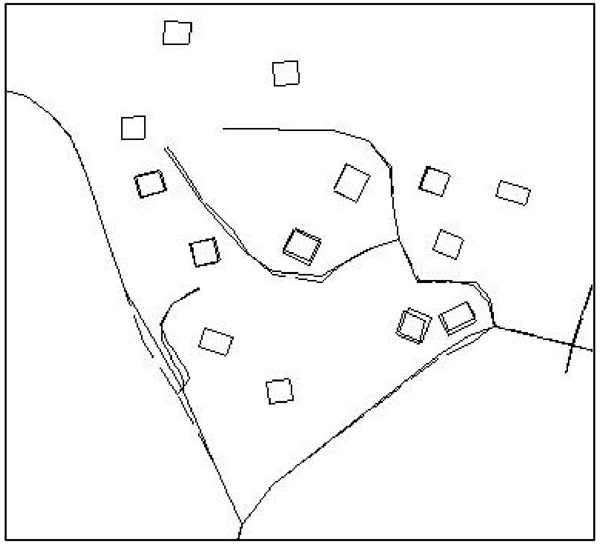
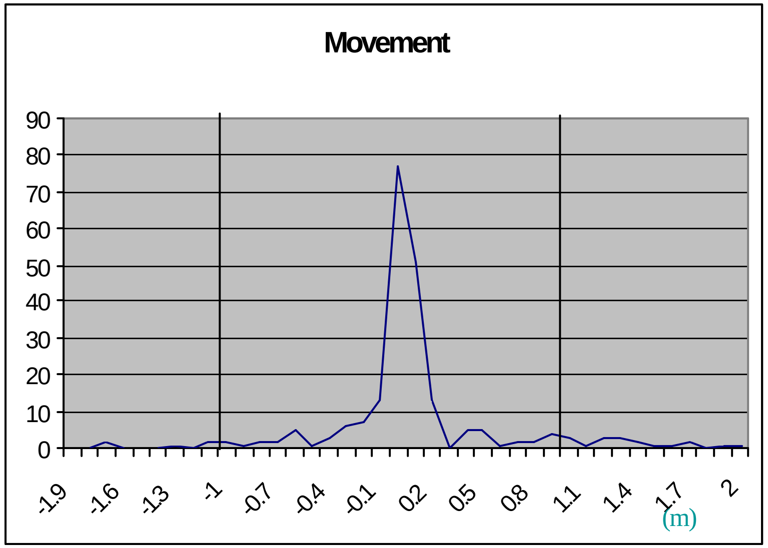
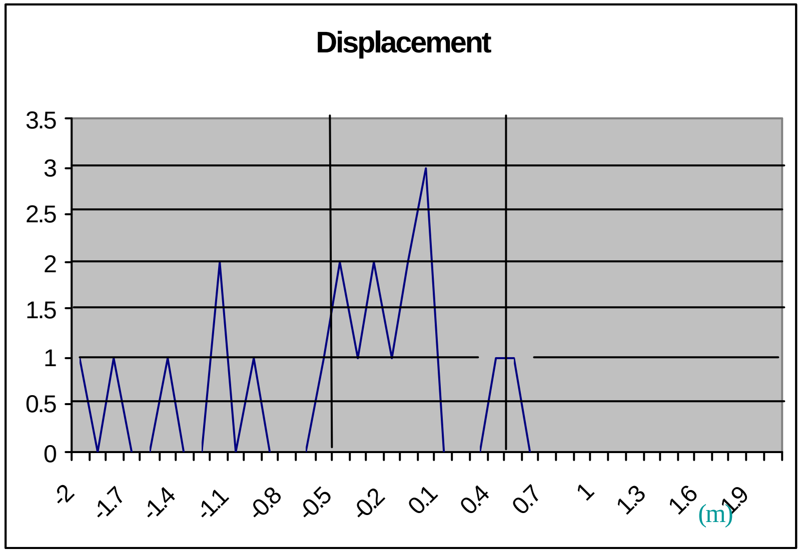
Result - test 1



Movement	Stiffness	Curvature	Segment length	Displacement
1 m	0.05 m	0.05 rad	0.5 m	0.5 m





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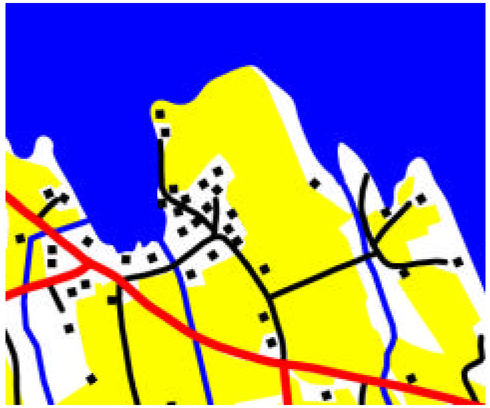
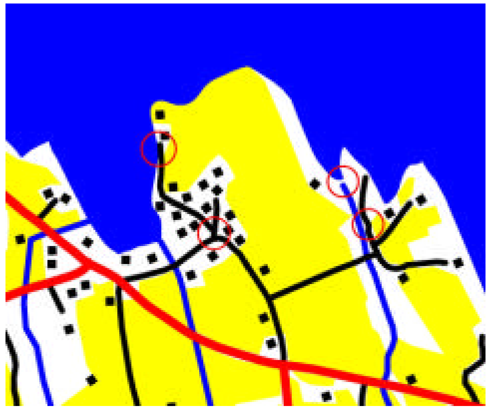
Result - test 2

Model generalisation



2856 points

Graphic generalisation



877 points

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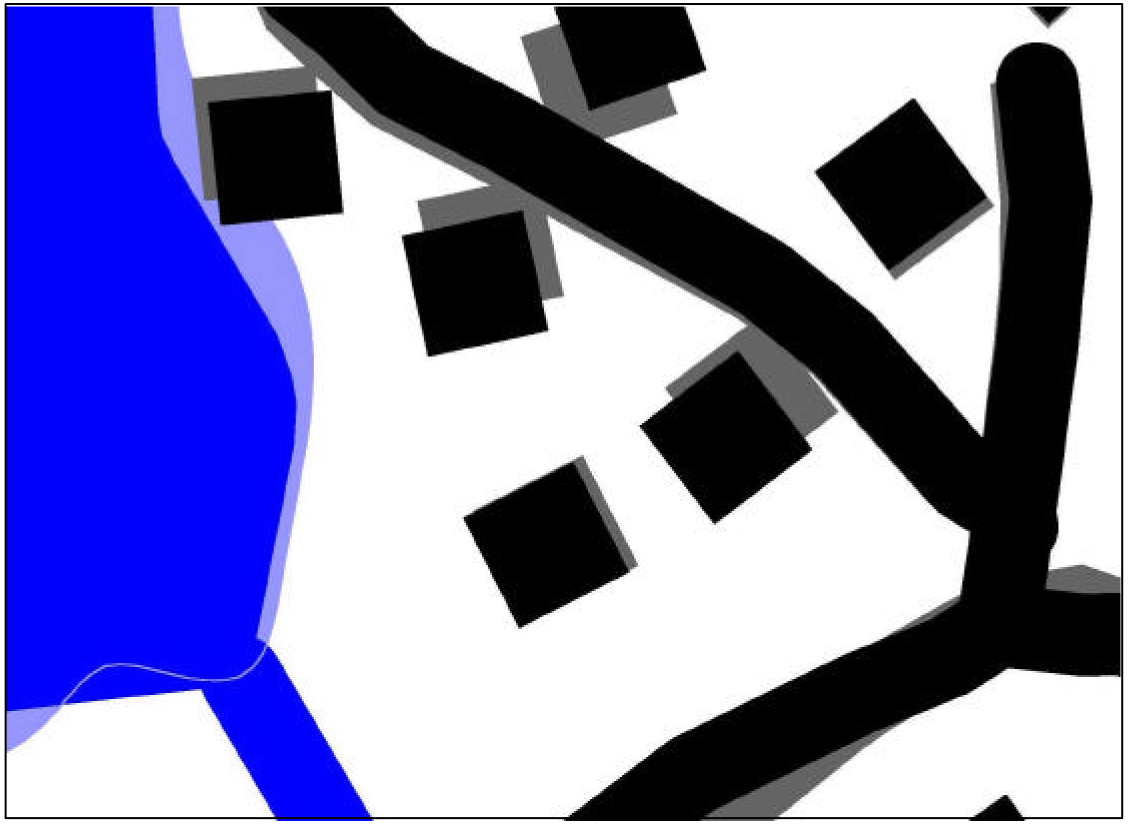
Result - test 2



Computational efficiency

Number of objects	Number of true points	Number of fictitious points	Total number of unknown	Number of constraints	Number of iteration in CGM	Running time (sec.)
49	606	595	1296	2325	301	2
51	2117	1727	4338	7063	123	7
101	2856	2408	6204	9744	246	9
118	2520	2089	5212	8479	352	13
245	4168	3348	8708	14105	348	20

Test indicates that the computational complexity is O(n*log n) time, where n is the number of points.

Weight-setting - test 3

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	Building-area	*Building-point*	*Major road*	*Minor road*	*Field*	*Power line*
Simplification (m)	*	*	1.0	1.0	1.0	1.0
Smoothing (m)	*	*	1.0	1.0	*	*
Exaggeration (m)	0.2	*	*	*	*	*
Displacement (m)	1.0	1.0	1.0	1.0	1.0	1.0
Movement (m)	8.0	8.0	4.0	8.0	8.0	8.0
Stiffness (m)	*	*	*	*	*	*
Curvature (rad)	*	*	0.05	0.1	0.1	0.05
Segment length (m)	*	*	1.0	2.0	2.0	2.0
Movement direction (m^2)	*	*	10.0	20.0	20.0	10.0
Crossing (m)	*	*	1.0	1.0	1.0	1.0

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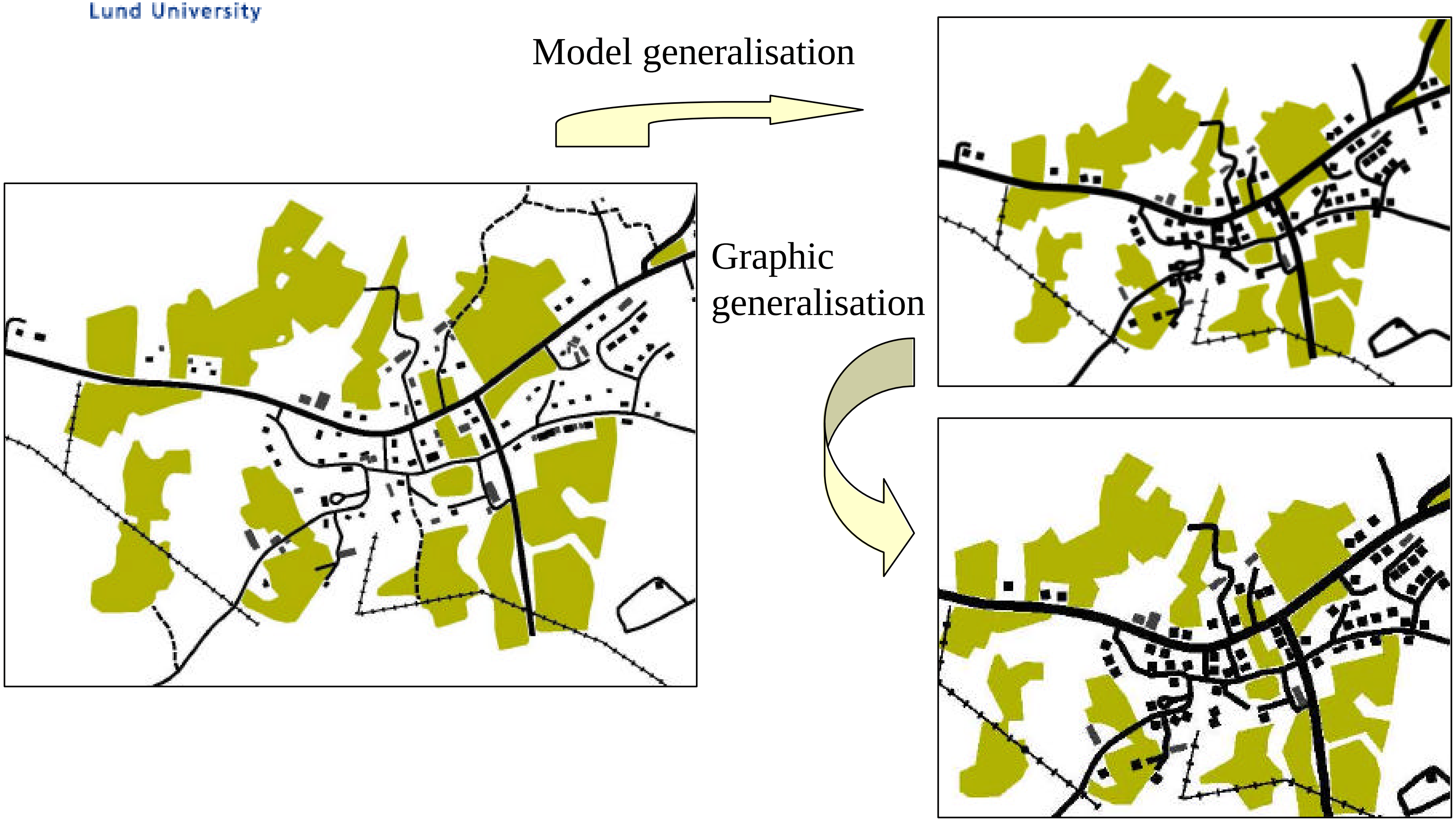
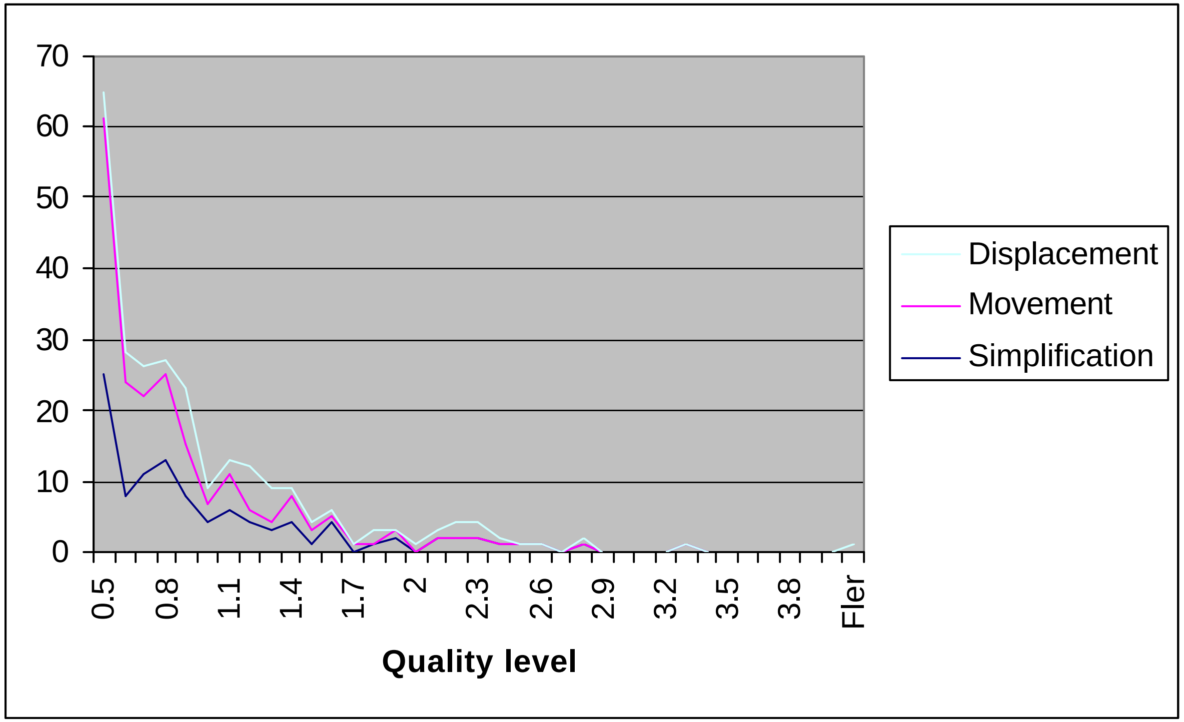
Result - test 3



Quality level - test 3

$$quality\ \ level_i = \sqrt{\mathrm{v}_i^2 \cdot \mathbf{P}_{i,i}}$$



Quality assessment - test 3

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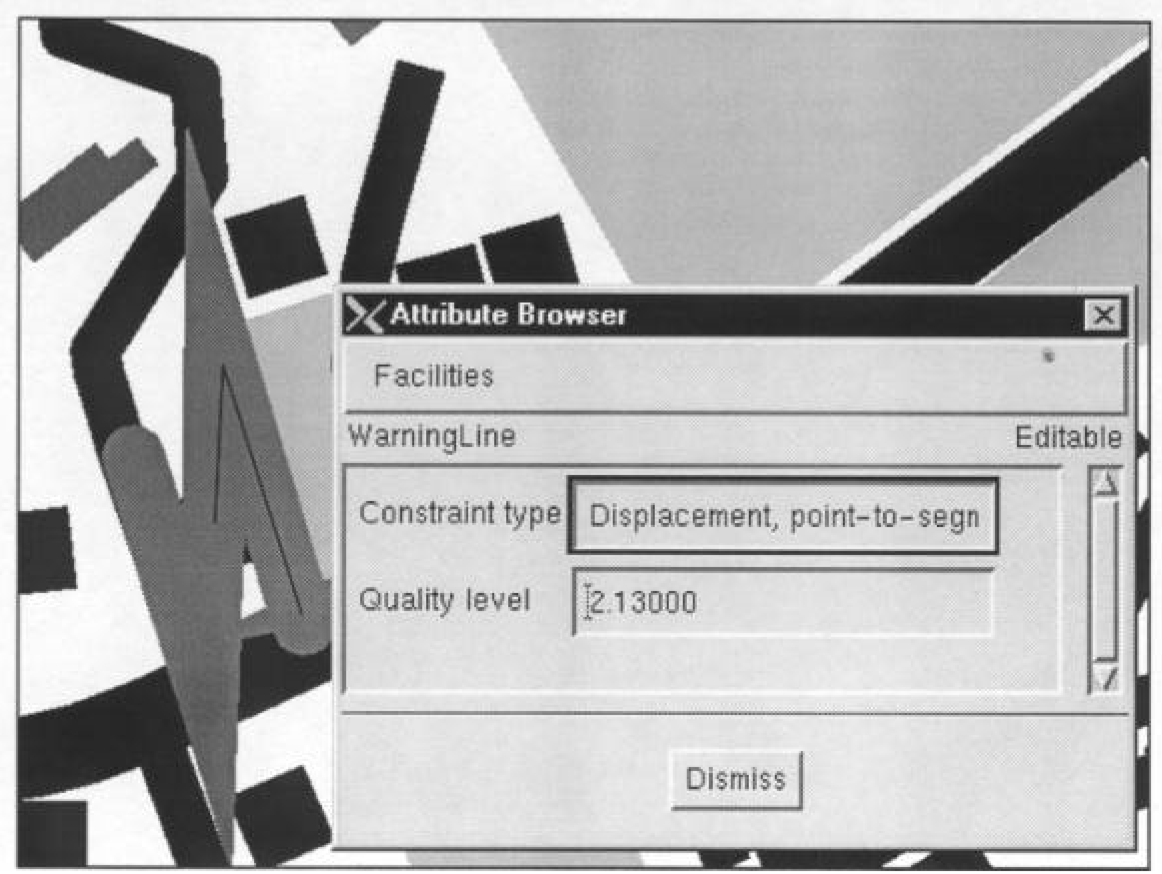

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Conclusions

Simultaneous graphic generalisation is useful for:

*as a pre-process prior to manual generalisation in map production

* for real-time generalisation of small areas