"A Methodology for the Assessment of Generalization Quality"

Andriani Skopeliti **Lysandros Tsoulos**

Cartography Laboratory, Faculty of Rural and Surveying Engineering
National Technical University of Athens
H. Polytechniou 9, 157 80 Zographou Campus, Athens, Greece
Tel: +30 +1+772-2730 Fax: +30+1+772-2734
email: askop@central.ntua.gr, lysandro@central.ntua.gr

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Abstract

Generalization quality assessment is a major issue in contemporary cartography. Besides the evaluation of generalization results, it supports the research for the automation of generalization. In this paper, two methods, based on structure and shape recognition, are elaborated: the parametric description of line shape and the partitioning of linear features into homogeneous segments. A number of quantitative measures for the assessment of line shape change due to generalization are identified. These measures provide an "a posteriori" evaluation of generalization alternatives, which result from the application of different generalization operators (i.e. simplification, smoothing), algorithms and tolerance values, onto a number of linear features of varying degree of complexity. In combination with existing quantitative measures for the horizontal position error, these measures constitute an efficient approach for the positional accuracy assessment at the individual object level. Specific tools, like graphs and tables, are proposed for the study of the positional accuracy aspect of any generalization schema. The application of this methodology will lead to the formulation of criteria, which can be used for the selection of the most suitable generalization solution and subsequently to knowledge acquisition and the development of a knowlwdge base to support automated map generalization.

1. Cartographic generalization and quality

Cartographic features' quality is influenced by a number of transformations throughout the map production process such as: generalization, projection change etc. Generalization may have unpredictable effects on the metric, topological and semantic aspect of a map. Each generalization operation influences certain elements of spatial data quality such as: positional accuracy, attribute accuracy, consistency and completeness (Muller, 1991). For example, displacement leads to lower accuracy, completeness is affected by selection and merging operations, some attributes may be lost through reclassification, consistency may be affected by uneven applications of spatial or temporal abstractions (Muller et al. 1995).

The assessment of spatial data quality is a major issue in contemporary cartography, influencing the decisions concerning the data fitness for use. Users want to minimize, control and quantify the effects of generalization on the data. Generalization quality assessment is also important for the evaluation of the results of automatic generalization and knowledge acquisition. Thus, the development of criteria, measures and evaluation methods is considered indispensable for generalization.

Measures for the evaluation of generalization alternatives can be characterized as quantitative and qualitative (Weibel, 1995; Ehrliholzer, 1995). Quantitative measures are further distinguished to global, geometrical, topological and software-related measures. Qualitative measures are based on the evaluation of expert's statements. Both quantitative and qualitative criteria should be developed to allow for the comparison of different generalization alternatives (among themselves and/or against a solution that is considered optimal), and eventually make it possible to judge and rank different solutions consistently (Weibel, 1996 p.66). According to Lagrange (1997) generalization quality assessment should be carried out for individual objects, groups of objects and entire map products in terms of their geometry, topology, semantics and aesthetics.

Criteria and methods (qualitative and quantitative) for the quality assessment of generalization methods, are largely missing (Weibel 1995); and thus generalization quality assessment remains an open research issue.

2. Positional accuracy of linear features

The majority of map features is either represented as lines (e.g. road centerlines, streams) or form polygons which are bounded by lines (e.g. administrative regions, soil polygons, forest stands) (Weibel, 1996). Positional accuracy is one of the main elements of linear features quality and it can be further analyzed to horizontal position accuracy and shape fidelity (European Committee for Standardization, 1996). Horizontal position accuracy (i.e. the ability to access the correct position) differs from shape accuracy (the ability to recognize the "true" shape of the object) (Muller et al., 1995). A number of generalization operators like simplification and smoothing influence positional accuracy.

Usually the results of a generalization operator are evaluated visually, mostly based on aesthetic criteria and less on quality assessment. However, generalization of digital data is an analytic procedure and cohesive criteria must be established for its evaluation. The evaluation of the positional accuracy aspect of cartographic generalization calls for the quantitative description of horizontal position and shape along with the development of measures for the assessment of their change.

Coordinates with respect to a reference system describe the horizontal position of spatial entities. Several measures have been identified (McMaster (1987, 1989), Jenks (1989) and Mustiere (1995)), for the horizontal position deviation between the original and the generalized line. A number of researchers carried out a systematic study on generalization results utilizing these measures (McMaster (1987, 1989); Jenks (1989); Joao (1995, 1998); Mustiere (1995)). On the same time, the description of the shape of linear features is an open research issue. If the shape of linear features is described in an objective way, its change due to cartographic generalization can be assessed. In the past a number of approaches for line shape description were suggested. Ruas and Lagrange (1995) classify techniques which rely on: directional changes such as Freeman chain encoding (Freeman, 1978), fractal analysis (Mandelbrot, 1967; Buttenfield, 1985), global characterizations (Jasinski, 1990; McMaster, 1987; Buttenfield, 1991; Bernhardt, 1992), detection of characteristic points (Thapa, 1989; Plazanet et al., 1995), techniques from artificial vision (Mokhtarian and Mackworth, 1992), description by means of mathematical formulation (Affholder, 1993) and frequency analysis, such as Fourrier (Fritsch and Lagrange, 1995). With regard to the preservation of shape during generalization, certain measures, which evaluate the degradation of the total sinuosity due to generalization have been identified by McMaster (1986), Jasinski (1990) and Buttenfield (1991). Plazanet (1996) identified evaluation measures especially for man-made features. The characterization and segmentation of cartographic lines is an important step towards this direction and falls into the field of Structure recognition (Weibel 1997).

According to Weibel (1997), structure recognition - that is the analysis of shape of map features - allows for the identification of the proper generalization approach and constitutes the basis for the selection, sequencing and parameterization of an appropriate set of generalization operators for a given line. However, all aspects of cartographic knowledge contribute to the generalization automation. According to Muller (1991) and Armstrong (1991), cartographic knowledge takes three different forms: geometrical, structural and procedural. Using linear entities described by their geometry (knowledge), structural knowledge is acquired through shape measurement techniques. Structural knowledge is combined with procedural knowledge, when cartographic generalization examples are retrieved from analog maps or are provided by expert cartographers.

In the following paragraphs, two methods for structure and shape recognition are elaborated: the parametric description of line shape and the partitioning of linear features into homogeneous segments. Based on the first method, special measures for the assessment of shape change due to generalization are identified. Utilizing these measures, which control shape fidelity along with the existing ones for horizontal position change, a complete approach to linear entities positional accuracy assessment can be formed (Figure 1).

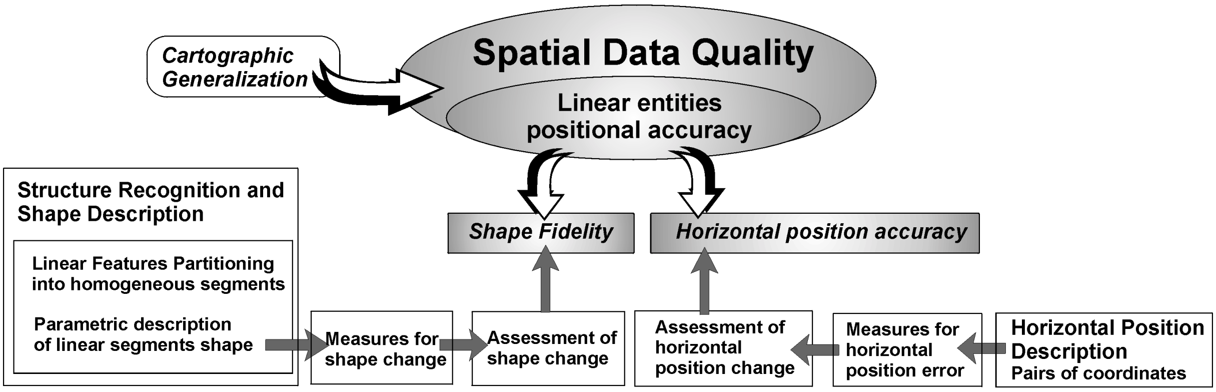

Figure 1. Positional accuracy assessment of linear features, utilizing structure recognition.

3. A methodology for structure and shape recognition

The methodology developed by the authors (Skopeliti and Tsoulos, 1999), aims at the parametric description of the shape of linear features. The fundamental concept in this approach, is the use of measures which are calculated at different resolution levels. This concept has been adopted by a considerable number of researchers, who followed different approaches for line shape description (Buttenfield, 1991; Bernhardt, 1992; Mokhtarian and Mackworth, 1992; Mandelbrot, 1967). In particular for angular measures computations, like in Carstensen (1990, p. 213), Thapa (1989) and (Plazanet 1996), lines are pre-processed in order to acquire a common resolution taking into account the source scale. In this way, angularity calculations are comparable with no bias due to the vertices spacing.

According to the proposed methodology, the shape of linear features is described by three parameters: the average magnitude angularity, the error variance and the ratio of length to the base line length. These parameters were selected from a broad set of parameters, utilizing Principal Components Analysis. The measures which were evaluated are mainly those proposed by Buttenfield and Bernhardt: fractal dimension, bandwidth, segmentation, error variance and concurrence (Buttenfield 1991); average angularity, average magnitude angularity, curvilinearity ratio, average vector displacement from baseline and average magnitude vector displacement from baseline (Bernhardt 1992). The ratio of the line length to the base line length is also introduced. This measure describes the deviation of the line from its simplest form, that is, the base line.

Based on the parametric description of line shape, segments can be allocated into similar shape groups through cluster analysis. The variables, which are selected with the application of Principal Components Analysis, exhibit low correlation between them and therefore a cluster analysis procedure can be applied.

In order to achieve a successful classification for linear entities, lines should be homogeneous along their entire length. The development of a segmentation methodology is therefore a prerequisite for a successful classification. Such a methodology should identify “homogeneous” parts and differentiate parts with varying degree of complexity. A considerable number of researchers have already focused on linear features segmentation such as Plazanet et al. (1995) using man-made features, Wang and Muller (1998) and recently Dutton (1999) using coastlines. The authors developed a methodology for natural linear features partitioning into homogeneous segments (Skopeliti and Tsoulos, 1999). This methodology is based on line shape assessment, utilizing fractal dimension and the above-mentioned methodology for line shape description.

The fractal dimension is utilized due to the fact that it exhibits certain advantages over other parameters which describe line complexity. Along with its value, an indicator of its ability to describe line character is provided. Moreover, it is a global parameter by definition, whereas others-such as average angularity-summarize local measures. Reliability criteria for the calculation of fractal dimension, have been formulated by Muller (1987) and Nakos (1990).

“Homogeneity” refers to the existence of similar characteristics along a line segment. In order to achieve segmentation, a homogeneity criterion is essential. When utilizing the parametric description of line shape, such a criterion can be formed with the utilization of the parameters results and a range of values. When the nature of lines in a specific data set is unknown, it is impossible to set “a priori” the critical values of the parameters, which imply a change in line character. The purpose of the approach elaborated here, is the identification of all of the self–similar segments existing at the line to be segmented and the selection of those resulting to its partitioning into segments with different character. The criterion ensuring the ability of the fractal dimension to describe the shape of a line, allows for the identification of the starting and the ending vertex of the self-similar segments.

The segmentation procedure is implemented in the following phases:

- Self - similar segments are identified along the line to be segmented
- Self - similar segments clustering: Self - similar segments, which are identified along the line to be segmented, are numerous and overlapping. They are described by the starting vertex, the ending vertex and the fractal dimension. In order to locate the parts along the line where spatial concentration (vertices contingency) and shape similarity (fractal dimension values proximity) exist, cluster analysis is utilized. The maximum value of clusters, which is essential for the application of hierarchical cluster analysis, is identified using an empirical index equal to the ratio of the length of the line to be segmented to the length of the shortest self-similar segment. When the clustering of self–similar segments is completed, a representative segment must be extracted from each cluster. The representative segment for each cluster is considered the one with fractal dimension value closer to the average value of the cluster.
- Preliminary segmentation: Representative segments may overlap. In order to avoid overlapping, the measures for the parametric line shape description are calculated and cluster analysis is performed. In this phase the methodology for the parametric line shape description is used, since it is not possible to determine with the use of fractal dimension only, whether the overlapping segments share the same degree of complexity. On the other hand, the use of the parametric description of line shape in conjunction with cluster analysis provide information on the segments' similarities. According to cluster analysis results, the following guidelines, characterized as Segment Management Rule I [SMR-I], are applied:
  - Overlapping segments belonging to the same cluster are joined,
  - A segment overlapping two segments belonging to different clusters is rejected,
  - A segment belonging in part to a segment of a different cluster is rejected.
- Segmentation refinement: The preliminary segmentation results in non-continuous line coverage, since only self-similar areas are identified. In order to acquire a continuous coverage, the segments located in-between the self-similar segments are added to the segment list. These segments are either non self-similar or too short to allow for the calculation of the fractal dimension. However, a segment should exist in the final segment list, if it differs from its contiguous ones. This condition is checked through the calculation of the measures for line shape description and the application of cluster analysis. A self-similar segment and a non-self similar segment or a “short” one belonging to the same cluster, can be joined if they result in a self-similar segment. This way the self–similar areas identified initially are not influenced.
- Final segmentation and segments clustering: When the segmentation process is completed, the parameters describing line shape are calculated for the final segments and cluster analysis is carried out for segment grouping.

The result of this procedure is the partitioning of the line in homogeneous segments as well as the grouping of those segments in similar shape clusters. An example of the application of this methodology is shown in Figure 2.

4. Consequences of generalization on the positional accuracy of linear features

In this paragraph, a number of quantitative measures for the assessment of line shape change due to generalization are identified. These measures provide an “a posteriori” evaluation (Weibel 1995) for the comparison of generalization alternatives. In combination with existing quantitative measures for the assessment of horizontal position change, these measures constitute an efficient solution for the assessment of the positional accuracy at the individual segment level.

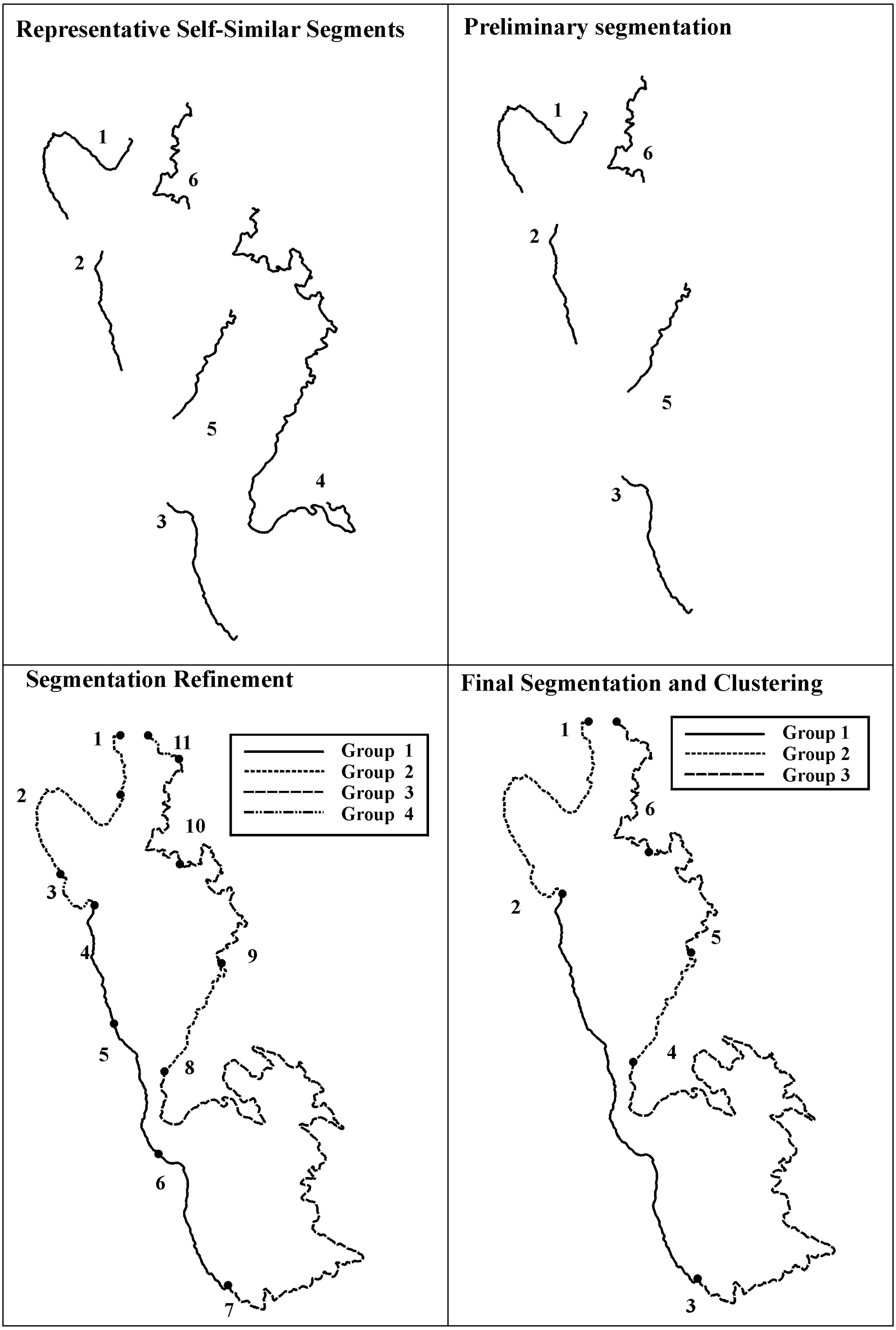

Figure 2. Application of the methodology for linear features segmentation

4.1 Influence on line shape

Generalization operators (such as simplification and smoothing), which influence positional accuracy, are implemented by special algorithms. The degree of an algorithm's influence on line shape is determined by the value of the tolerance used. Although the selection of a tolerance value sets an upper limit to the horizontal position error, the consequences to the shape of the cartographic line remain unknown.

The group of the above-mentioned parameters can describe the shape of any line, original or generalized. In addition, through the parametric description of line shape, the shape change of a linear feature in comparison with the original line can be also assessed. In cluster analysis, the distance between two lines in the parameters' space implies similarity. The distance between the original and the generalized line in the parameters' space describes shape change. This is a quantitative assessment of shape change due to generalization. The average of the shape change values for the lines, which make up a group, represents the average line shape change for this group.

When a generalization schema is applied to all line segments, generalized segments are grouped in different clusters than the original ones. The allocation of lines in the new groups is studied in order to draw conclusions on the trends of the modification of lines shape. Through the application of cluster analysis, which enables the comparison of generalized segments between them and the initial ones, a qualitative assessment of the line shape change is achieved. The degree of similarity is expressed through the classification results. The exploitation of the cluster analysis technique leads to the transformation from the quantitative to the ordinal scale. The examination of the generalized lines clustering results leads to conclusions, which vary with the method used.

- When non-hierarchical cluster analysis is applied utilizing the centers of the original lines groups, the results describe the generalized lines similarity to the original ones. For example, allocation of the line to the initial group of smoother lines indicates that the simplification algorithms create less complex lines.
- When hierarchical cluster analysis is applied, the generalized lines are clustered independently and the results express the similarity between them. A "good" generalization solution is the one preserving the number and the composition of the initial groups.

The change in the groups' composition due to generalization shows that all line categories should not undergo the same generalization schema. The application of an inappropriate generalization schema to a group of lines may alter their characteristics so drastically that they can no longer form a distinctive group. On the contrary, if the generalized lines, which were initially considered as a group, form a new group, it means that they have changed similarly. This is true only when the generalization schema preserves the shape of this line group.

4.2 Influence on the horizontal position

The horizontal position deviation between the original and the generalized line, can be assessed by the following measures: a. the average Euclidean distance from the original to the generalized line or from the generalized to the original line, b. the Hausdorff distance (Abbas et al., 1995), and c. the ratio of the area between the original and the generalized line to the length of the original line (McMaster, 1987). The average change of the horizontal position for a group of lines is equal to the average of the values calculated for the individual line segments.

4.3 Study of the positional accuracy

Based on the assessment of the line shape change, the influence of the generalization elements (generalization operators, algorithms, tolerance values) can be studied. A number of issues can be addressed as the relation between line shape change and the generalization elements. Moreover, if groups of lines of different complexity exist in the data set, an in depth study can be carried out. The relation between line complexity and line shape change due to different generalization operators, algorithms and tolerances values can be examined. Likewise, utilizing the measures for the horizontal position error, several questions can be answered as the relation between the horizontal position error and the generalization elements (operators, algorithms and tolerance values). More specifically the following tools can be used:

- **Horizontal position accuracy** can be studied at the operator level, where the average horizontal position error values for different algorithms are compared or at the algorithm level, where the average horizontal position error values for groups of lines of different complexity are compared. The creation of graphs and tables with the items shown in Table 1 is proposed (Table 1).

Study Level	Item 1	Item 2
1. Operator: Comparison between algorithms	Average horizontal position error values *for each algorithm*	Tolerance values
2. Algorithm: Comparison between group of lines of different shape	Average horizontal position error values *for each group of lines*	Tolerance values

Table 1. Study of the horizontal position accuracy

- **Shape preservation** can be studied at the operator level, where the average shape change values for different algorithms are compared or at the algorithm level, where the average shape change values for groups of lines of different complexity are compared. The creation of graphs and tables with the items shown in Table 2 is proposed. In addition, the construction of a table showing the clustering results (hierarchical and non-hierarchical) of different generalization schemas (different algorithms and a range of tolerance values) and the comparison with the initial segments clustering is considered useful.

Study Level	Item 1	Item 2
1. Operator: Comparison between algorithms	Average of shape change measure values *for each algorithm*	Tolerance values
2. Algorithm: Comparison between group of lines of different shape	Average of shape change measure values *for each group of lines*	Tolerance values

Table 2. Study of the shape preservation

- For the study of the **positional accuracy**, the creation of graphs and tables with the items shown in Table 3 is proposed.

Study Level	Item 1	Item 2
1. Operator: Comparison between algorithms	Average of shape change measure values *for each algorithm*	Average horizontal position error values *for each algorithm*
2. Algorithm: Comparison between group of lines of different shape	Average of shape change measure values *for each group of lines*	Average horizontal position error values *for each group of lines*

Table 3. Study of the positional accuracy.

4.3.1 Application

The above-described methodology for the assessment of generalization consequences to the horizontal position and the shape of linear features, has been applied in the framework of a pilot project implemented in the following stages:

i. Linear features partitioning in homogeneous segments (see Figure 2),

ii. Linear segments character description, utilizing a group of parameters and subsequent clustering in groups with similar shape (see Figure 2),

iii. Implementation of several generalization solutions through the application of a number of different simplification algorithms and tolerance values,

iv. Assessment of the positional accuracy of generalization results with regard to shape and horizontal position change.

Several examples of the methodology implementation follow:

a. Comparison of shape change caused by simplification algorithms

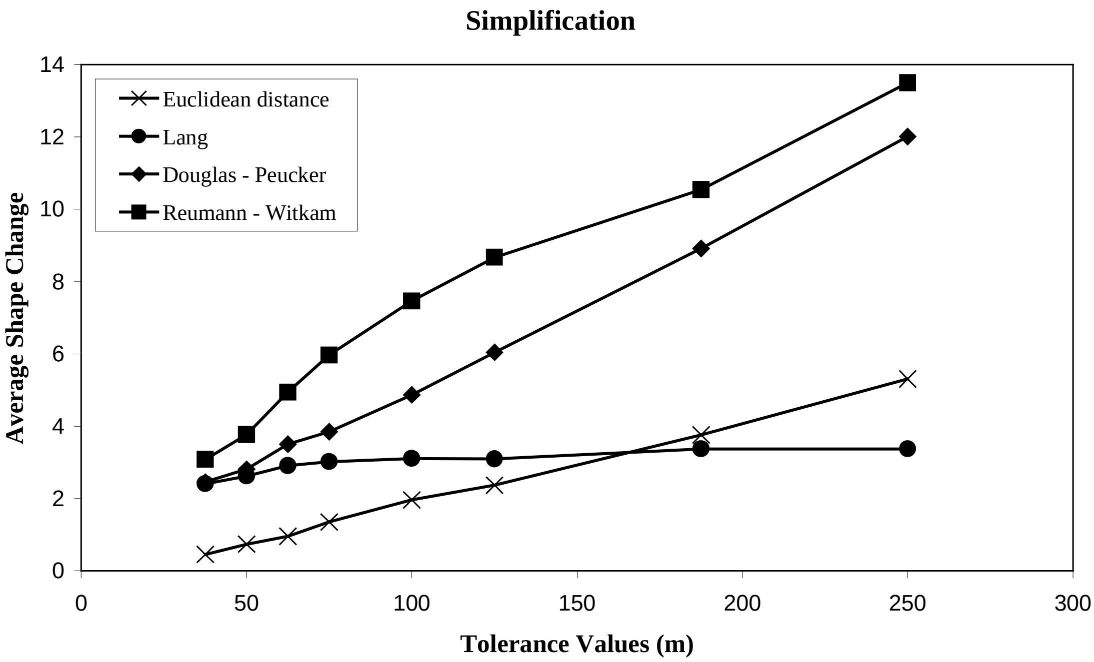

Figure 3. Average shape change caused by different simplification algorithms for a range of tolerance values

b. Algorithms influence on line groups of different shape

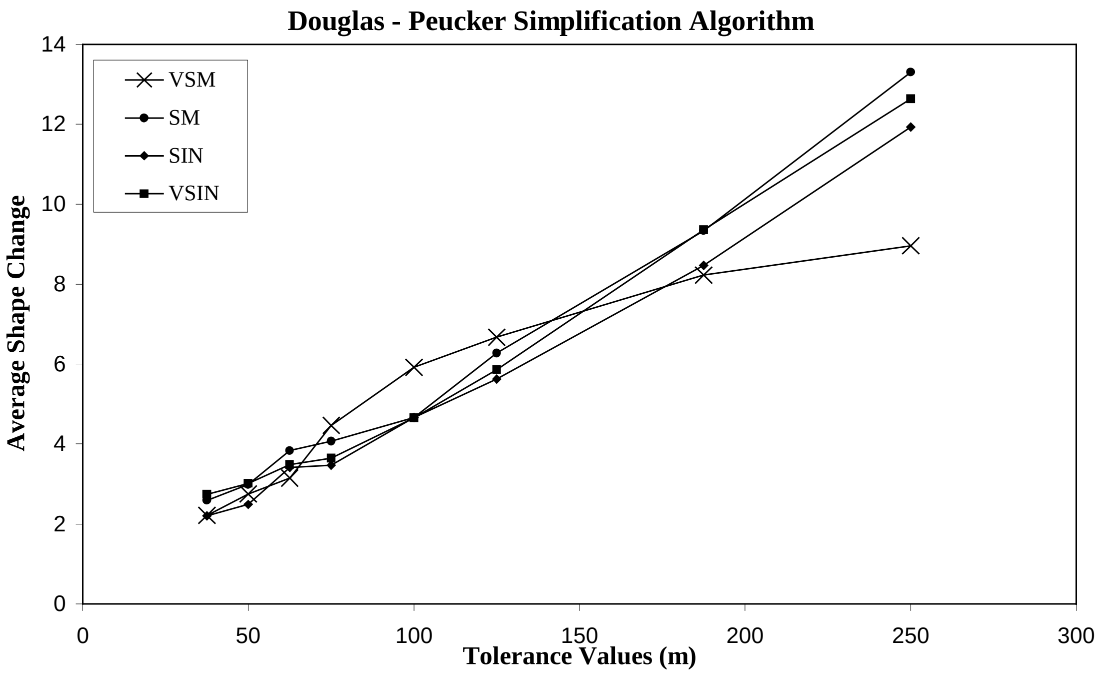

Figure 4. Average shape change caused by the Douglas – Peucker simplification algorithm and a range of tolerance values on groups of lines of different shape (VSIN - very sinuous, SIN - sinuous, SM - smooth, VSM - very smooth)).

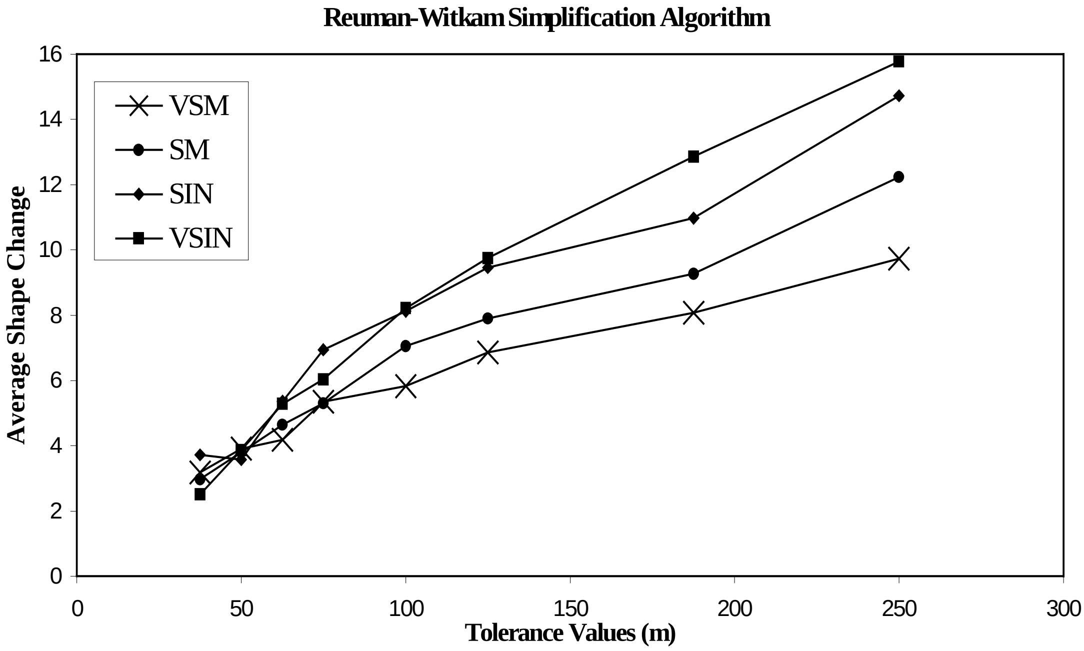

Figure 5. Average shape change caused by the Reuman - Witkam simplification algorithm and a range of tolerance values on groups of lines of different shape (VSIN - very sinuous, SIN - sinuous, SM - smooth, VSM - very smooth)

c. <u>Comparison of the clustering results of different generalization schemas</u>

Original			**Simplified**					
Scale x 1000		100	200	500	1000	200	500	1000
Line Code	Group		Hierarchical			Non-hierarchical		
4	VSM	1	1	1	1	1	1	1
9		1	1	1	1	1	1	1
14		1	1	2	1	1	1	1
1	SM	2	2	2	1	2	1	1
7		2	2	2	1	1	1	1
10		2	2	2	1	1	1	1
13		2	2	2	1	2	2	1
16		2	2	2	1	2	2	1
2	SIN	3	3	3	2	2	2	2
3		3	3	3	2	3	3	2
6		3	3	3	2	3	2	2
8		3	4	4	2	3	3	2
12		3	3	3	2	3	3	2
5	VSIN	4	4	4	3	4	3	3
11		4	4	4	3	4	3	3
15		4	4	4	3	3	3	3
17		4	4	4	3	4	4	3
18		4	4	4	2	4	3	2
Nu. of Groups		4	4	4	3	4	4	3

Table 4. Hierarchical and Non-Hierarchical clustering results of simplified lines. (Douglas-Peucker algorithm for a number of scales (VSIN - very sinuous, SIN - sinuous, SM - smooth, VSM - very smooth)).

d. Horizontal Position Accuracy

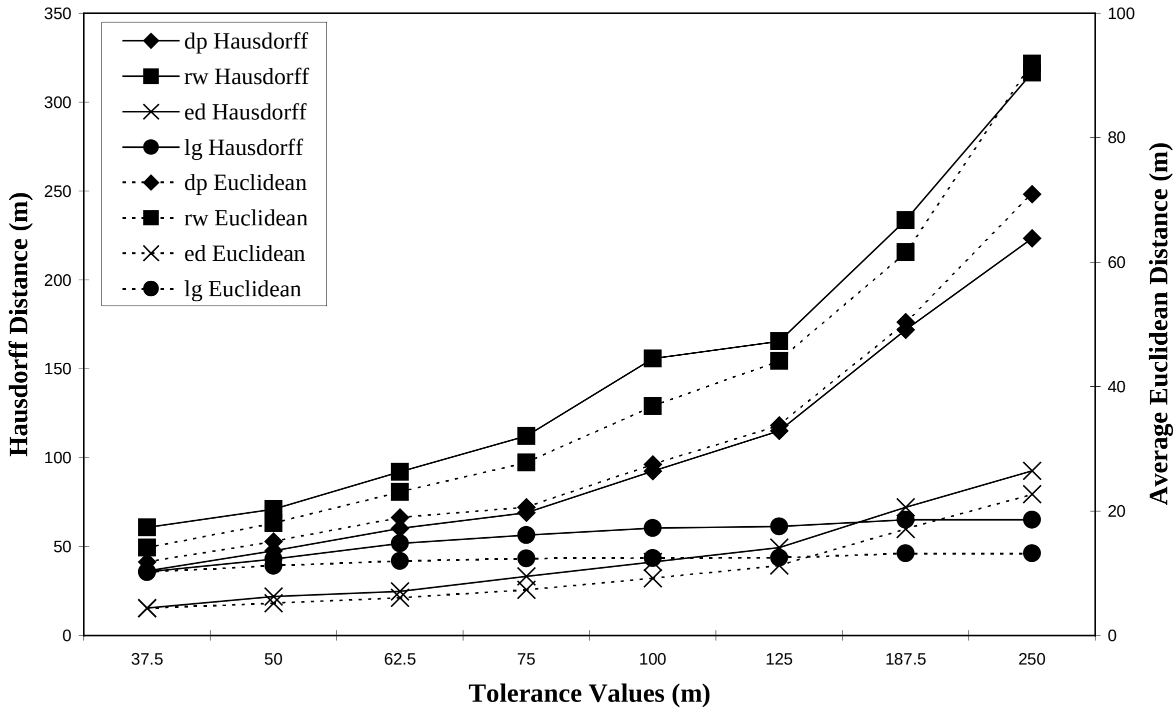

Figure 6. Horizontal position error, expressed by the Hausdorff distance and the Average Euclidean Distance, caused by different simplification algorithms and a range of tolerance values.

e. Overall Positional accuracy

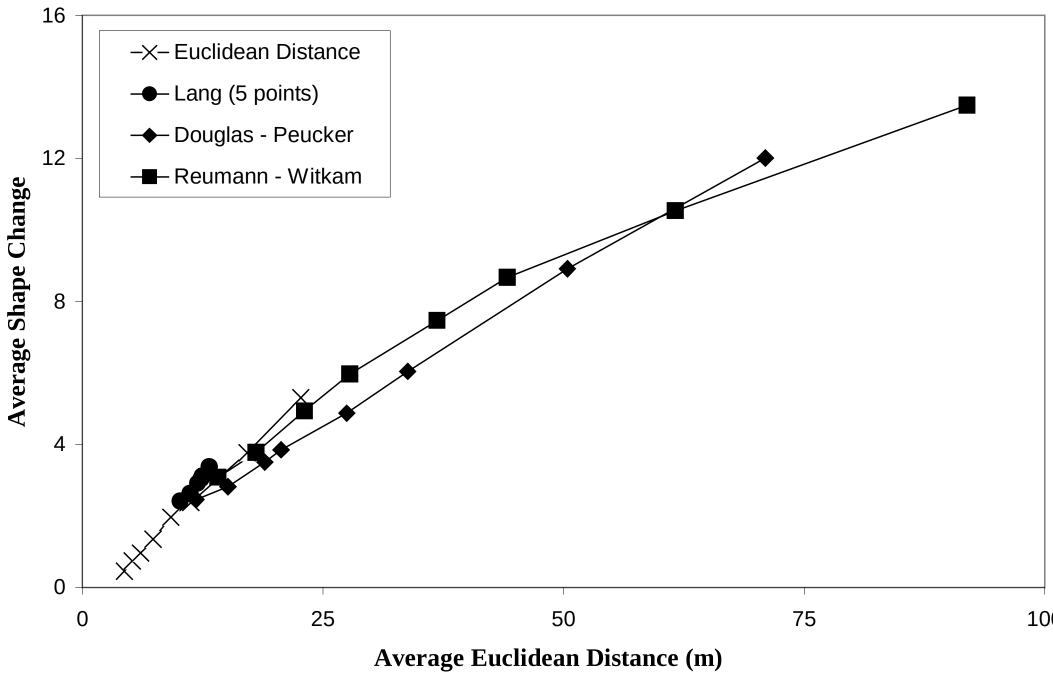

Figure 7. Average shape change and average Euclidean distance between the generalized and the original line caused by different simplification algorithms and a range of tolerance values.

5. Assessment of the generalization quality

5.1 Assessment tools

In order to judge the quality of the generalized data utilizing the positional accuracy measures, there is a need for the development of specifications and the identification of the appropriate constraints. *Assessment tools* can determine whether the relevant constraints are satisfied. *Measures* are needed to describe quantitatively the generalization effects. Specific *conditions* are checked utilizing measures' values and the post-generalization situation can *be assessed*.

When examining the line generalization quality at the line level, the following constraints can be identified: metric (avoid imperceptible crenulations, avoid self-coalescence, minimize shape distortion), topologic (avoid self-intersection) and gestalt (preserve original line character) (Weibel, 1996). Regarding the positional accuracy aspect of the generalization results, the following *constraints* can be considered: maintenance of shapes variety, preservation of the original characteristics of the lines, degree of simplification suitable for the new map scale, minimization of shape distortion and minimization of horizontal position error. These constraints belong to the metric, structural and gestalt categories. The constraint, which refers to the minimization of the line shape distortion, can be checked with the average shape change measure. Regarding the preservation of the original line character, criteria are based on information concerning the initial and the generalized data. For example, a basic characteristic of the generalized data is the ability to recognize the different line shapes as in the original data. In order to ensure this, the number of different shapes existing in the original data should be retained. Several measures can be used for these constraints: the number of the generalized line groups and the synthesis of the generalized lines groups. In order to check the horizontal position accuracy constraint, a condition can be formed using the horizontal position error values and the legibility threshold of the new scale.

These assessment tools are based on the preservation of certain properties of the initial data expressed by the constraints and thus the need for procedural knowledge to define "the correct generalization solution" diminishes. The degree, to which the constraints are satisfied, depends on the scale change and the scope of the new map. The authors carried out a more elaborated approach on the formation of assessment tools utilizing measures and constraints (Skopeliti and Tsoulos, 2001).

5.2 Knowledge acquisition

The second application refers to the correlation of the cartographic knowledge with the change of positional accuracy. This can be implemented using "correct generalization examples" and the measures of change of the two components of positional accuracy. Recent research focuses at the correlation of accepted generalization solutions (operator, algorithm tolerance values) with line shape, as it is described by a group of parameters. In particular, Weibel et al. (1995) apply a machine learning technique to derive prototype rules, which relate the tolerance values of the Lang simplification algorithm to groups of lines of different complexity. Line shape (structure recognition) is described by a group of parameters (utilizing cluster analysis several ranked classes per attribute are created). A user, who selects the Lang algorithm tolerance values in order to match manual generalizations, provides procedural knowledge. In addition, Lagrange et al. (2000) use neural networks to relate the tolerances values of the Gauss smoothing algorithm with groups of lines with different complexity. Line shape is described by a group of parameters and procedural knowledge is acquired in an interactive environment, through the identification of a range of tolerances for the smoothing algorithm, utilizing - as reference - manually generalized features. In this paper, a different approach is proposed. Measures, which describe the positional accuracy, are computed for manually generalized data or cartographically acceptable generalization results, which serve as exemplars. These measures are also computed for the "automatically" produced generalization results. Based on these measures, "exemplars" and "automated generalization" results can be compared quantitatively and related objectively without human intervention. The methodology for shape and structure recognition is applied to the original data (Skopeliti and Tsoulos, 1999). Thus structural knowledge is described formally and lines are clustered into similar shape groups. Utilizing machine-learning techniques, structural knowledge can be related to the automatic generalization solution, which is closer to the exemplars and procedural knowledge can be acquired.

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