MODELLING CONSTRAINTS FOR POLYGON GENERALIZATION

Martin Galanda

Department of Geography, University of Zurich Winterthurerstr. 190, 8057 Zurich, Switzerland mgalanda@geo.unizh.ch

ABSTRACT

Generalization constraints, that is design specifications to which map generalization should adhere, have been established in the past as concept for controlling automated generalization processes. From the perspective of building a multi-agent-system (MAS), this paper proposes a set of constraints for the generalization of polygonal subdivisions (polygon generalization). Besides an inventory of constraints for polygon generalization, the paper provides methods for the evaluation of the individual constraints' satisfaction, plans (algorithms and parameters) for improving the satisfaction of constraints and importance constraints that allow compromise between different constraints. Hence, this set is well qualified to control automated polygon generalization. In other words, these constraints can be used to detect conflicts, to trigger generalization algorithms, to evaluate accomplished solutions and to identify the best compromise among several solutions. This comprehensive set of constraints can build the foundation of any automated generalization process of polygonal subdivisions. So, our future research will focus on the integration of this set of constraints and constraint evaluation methods into an agent-based framework for automated polygon generalization.

Keywords polygon generalization, generalization constraints, measures, plans, importance;

1 Introduction

Automated map generalization remains one of the most challenging research topics in cartography and geographic information science since several key generalization tasks are not yet satisfactorily solved. Recent generalization projects – compare, for instance, Ruas (1999), Harrie (1999) and Barrault et al. (2001) – proved the potential of constraint based approaches to map generalization, that is, constraints control the map generalization process. A constraint denotes "a design specification to which solutions should adhere" (Weibel and Dutton 1998, p. 215), that allows a consider about both the user's needs and mapping principles such as minimal dimensions.

Polygonal subdivisions, i.e. categorical data (e.g. land use, geology, administrative units etc.) represented by vectors, are the second key ingredient of this paper and are commonly used in geographic information systems and thematic maps, respectively. Research concerning its (automated) coherent generalization still remains in the early stages (Galanda and Weibel 2002b). Hence, this paper concentrates on the generalization of polygonal subdivisions (polygon generalization). What is missing in polygon generalization is a comprehensive system that combines different existing methods of polygon generalization such as algorithms, measures and constraints into a coherent generalization process. Thus, previous research by the authors has dealt with the development of an agent-based framework for automated polygon generalization (Galanda and Weibel 2002a). The general approach followed in this framework reverts to concepts and terminology developed in the AGENT project (Barrault et al. 2001) and by Ruas (1999) which aimed at the development of self-evaluating generalization methodologies using multi-agent systems (MAS). This approach is based on constraints, that is, constraints are used to detect conflicts, to control the resolution of conflicts and to evaluate and compare accomplished solutions (Weibel 1996, Ruas 1998).

Previous work such as Weibel (1996), Peter and Weibel (1999) and Edwardes and Mackaness (2000) discussed constraints for polygon generalization mainly on a conceptual level. Starting from their work a basic set of constraints for polygon generalization should be established. The novelty of this paper lies in enriching this set in such a way that the constraints of the set could be easily applied

to automated polygon generalization, that is, these constraints should have the potential to control such an agent-based generalization process. The resulting requirements are specified in the next section together with a generic discussion on generalization constraints. The remaining sections deal exclusively with constraints for automated polygon generalization and their modelling. Altogether, the paper provides the basis for the integration of this set of constraints into the framework for agent-based automated polygon generalization outlined in Galanda and Weibel (2002a).

2 Generalization constraints

The concept of constraints was originally transferred from computer science to map generalization by Beard (1991) and later on emphasized by, among others, Mackaness (1995), Weibel (1996), Weibel and Dutton (1998), Ruas (1999) and Barrault et al. (2001). This section intends to provide an overview of both the basic concept of constraints in map generalization and of different taxonomies of constraints relevant to this work.

2.1 Concept of generalization constraints

The concept of constraints used in this paper owes to previous research conducted by Ruas (1998, 1999) and in the AGENT project (Barrault et al. 2001). A constraint designates a final product specification on a certain property of an object that should be respected by an appropriate generalization. Constraints are often implemented as functions of comparison. For instance, the size of a forest polygon should be greater than $1000m^2$. In comparison to rules, constraints are neither bound to a single condition – in fact constraints are often related to a synthesis of conditions (Ruas and Plazanet 1996) – nor to a particular action (Beard 1991). Every constraint is linked to the following values and methods (Ruas 1998, 1999, Barrault et al. 2001) – their interdependencies are shown in Figure 1:

- a *goal value* that defines the value an object should at least reach or maintain during generalization according to a certain constraint. In other words, goal values are thresholds that objects should respect in order to satisfy a certain constraint.
- a *measure*, i.e. a method for computing the current value of the property that the constraint refers to.
- an *evaluation method* that examines the compliance of an object to a certain constraint, that is, it determines the satisfaction of the constraint according to this object. A so-called severity value describes the level of satisfaction, that is, the discrepancy between the measure's result and the goal value previously defined. The satisfaction of constraints is standardized to a range from 1(very bad) over 2(bad), 3(medium) and 4(good) to 5(perfect) in order to allow the comparison of the severity of different constraints.
- a list of plans $[plan_1, plan_2, ..., plan_n]$. A plan designates a list of cartographic algorithms and corresponding parameters that are suited to improve the constraint's satisfaction. In consideration of the severity and the specific situation the same constraint may propose different lists of plans.
- an *importance value* denotes the relative importance of constraints, that is, it represents the importance of a constraint to the quality of a generalization result, in comparison to other constraints. The importance value is essential to compromise between different constraints attached to the same object and to validate the affect of a generalization operation.
- a *priority value*. This describes the priority of treatment of a constraint in the generalization process, that is, it defines which constraint should be solved first. Note that, constraints of high importance need not to have a high priority as well.

Consequently, every constraint of the aspired set must hold a goal value, a measure, an evaluation method, a list of plans, an importance value and a priority value, in order to meet the requirements of agent-based generalization.



Figure 1: The values and constraints attached to a constraint and their interdependencies.

2.2 Taxonomies of generalization constraints

Since there seems to be no agreement on taxonomies of generalization constraints in the 'generalization community' – compare, for instance, those taxonomies discussed in Weibel (1996), Harrie (1999) and Ruas (1999) – this section provides a brief overview of those taxonomies of constraints used throughout this paper. The discussed taxonomies accomodate the fact that every generalization constraint

- is bound to a specific spatial level of a polygonal subdivision.
- is related to a certain aspect of a polygonal subdivision, namely either its graphical appearance, its underlying topology, its spatial and semantic structure or its generalization.
- plays a specific role in the generalization process.

Spatial levels. A generalization constraint is always evaluated with respect to a specific cartographic object. Cartographic objects in polygonal subdivisions occur at different so-called spatial levels. The concept of spatial levels in map generalization deals with the organization of map space into spatial entities. Ruas (1999, 1998) and Mustière and Moulin (2002) demonstrated that the consideration of spatial context in map generalization allows better results to be achieved more efficiently. So every generalization constraint, measure, plan and algorithm is bound to a spatial level, that is, every constraint is linked to the properties of such a level (Regnauld 2001). For polygon generalization Galanda and Weibel (2002a) suggested the use of four different spatial levels, namely

- a *map level* referring to the entire polygonal subdivision,
- a *group* level dealing with groups of polygons such as all polygons that belong to the same category or cluster,
- a *polygon* level concerning individual polygons, and
- a *line* level designated to polygon boundaries.

With respect to a more general organization of map space – compare Ruas (1999), Barrault et al. (2001) – the map level corresponds to a macro level, the group level to a meso level and both the polygon and line level to a micro level.

Similar aspects. Alternatively, constraints may be subdivided into four groups of constraints that concern similar aspects of a data set (Dutton et al. 1998, Weibel and Dutton 1998, Ruas 1999):

- *Metric (or graphical) constraints* translate human limits of perception (Malič 1998) into metric perceptibility thresholds (minimal dimensions) of map objects. The specification of corresponding goal values depends on the target scale, the output medium (e.g. paper or screen), the map purpose and the cartographic representation (e.g. contours or filled contours). Minimal dimensions of polygonal subdivisions are among others discussed in SSC (2002) and Spiess (2002).
- *Topological constraints* ensure that the topological structure of a polygonal subdivision is maintained or modified consistently (Dutton et al. 1998, Ruas 1999). For instance, self-intersections of a polygon boundary or any intersection of two polygon boundaries must be avoided.
- Structural constraints intend to preserve the spatial and semantic structure of a data set in consideration of aesthetics and visual balance. With respect to aesthetics and visual balance some authors, such as Weibel (1996) and Dutton et al. (1998), proposed a further distinction of so-called gestalt constraints. This work abstains from this distinction. Structural constraints are dealt with on the level of an individual polygon (e.g. shape characteristics of a polygon), a group of polygons (e.g. spatial alignment of objects, size ratios of objects) and the whole data set (e.g. relative areas of categories).
- *Procedural constraints* relate to the generalization process itself. So, they influence the sequence of generalization operations and the selection of suitable algorithms and parameters, respectively. Such a constraint may specify a threshold of semantic similarity for merging polygons, for instance, in the course of an elimination or aggregation operation.

Generalization process. Another taxonomy looks at the role of constraints in the generalization process. Hence, constraints are divided into offensive and defensive constraints (Galanda and Weibel 2002a). Offensive constraints define a need for generalization and every appropriate generalization of a data set must meet them, respectively. Thus, the violation of such a constraint always leads either to a spatial or semantic transformation of a data set (e.g. in the case of the violation of a metric constraint) or to the rejection of a generalized data set (e.g. in the case of the violation of a topological constraint). Defensive constraints denote such constraints that need not to be met imperatively but strive for optimal compliance. They are flexible, while offensive constraints are strict. For instance, shape distortion or changes in size ratios should be kept as low as possible. While they are not imperative, defensive constraints may help to identify the best solution among different possible solutions which equally satisfy all offensive constraints. With respect to the generalization process, Ruas (1999) proposed a slightly different distinction, namely constraints of maintenance and of generalization. Constraints of maintenance should preserve a property as faithfully as possible but, in contrast to defensive constraints, they can be either strict (e.g. topology) or flexible (e.g. polygon shape). Constraints of generalization are those constraints that must be respected, such as the minimal size of a polygon.

3 Constraints for polygon generalization

While the previous section dealt with constraint on a general level this section presents a conceptual discussion of constraints for polygon generalization. Constraints are organized, according to the taxonomy of 'similar aspects', into groups of metric, topological, structural and procedural constraints (cf. section 2.2). Additionally, the spatial level that a constraint refers to is annotated in parentheses behind every listed constraint. Subsequent sections examine methods for both the evaluation of the satisfaction of constraints (section 4) and the coherence of generalization operators and constraints' satisfaction (section 5).

As cartographic knowledge related to map generalization in general (Weibel et al. 1995, Müller et al. 1995), constraints for polygon generalization may be extracted from

- interviews with cartographic experts (Schylberg 1993, Kilpeläinen 2000),
- existing map series (Edwardes and Mackaness 2000),
- *textbooks* of (thematic) cartography (Imhof 1972, Dent 1990, Slocum 1999) or map generalization (SSC 2002), and
- guidelines of mapping agencies (McMaster 1991, Landesvermessungsamt Nordrhein-Westfalen 1993).

With respect to polygon generalization these sources often focus either on very specific topics such as the elimination of an individual polygon (Schylberg 1993, Kilpeläinen 2000) or the generalization of a single category such as 'forest' as a layer of a topographic map (Imhof 1972, Arnberger 1993, SSC 2002). Hence, the inventory of constraints for polygon generalization provided here is based on both constraints derived from the sources listed above and those constraints proposed initially by Weibel (1996).

3.1 Metric constraints

M1 Consecutive vertex distance (line level). Consecutive vertices of a polygon boundary should be separated by a minimum distance at least. This constraint intends not to trigger generalization but to speed up the generalization process by removing redundant vertices from a polygon boundary while maintaining the polygon shape as faithfully as possible (Visvalingam and Williamson 1995).

M2 Outline granularity (line level). Imperceptible crenulations of a polygon boundary must be eliminated (cf. Figure 2).

M3 Distance between boundary points (polygon level). Any non consecutive points of a polygon geometry should be separated by a minimum distance at least.

M4 Minimal area (polygon level). All polygon objects should have at least a minimal area for the given target scale. In general, objects should "be large enough for the reader to see and differentiate areal patterns" (Dent 1990, p. 152).

M5 Respect spatial context (polygon and group level). Individual polygons and groups of polygons should respect their spatial context in conflict resolution. In other words, this constraint prevents the creation of new conflicts between generalized polygons or groups of polygons and other polygons that are not generalized at the same time. For instance, a group of disjoint island polygons should respect their spatial context, that is, the polygons that embed them (cf. Figure 3).

M6 Object separation (group level). The distance between two disjoint polygons should be not less than a minimum distance.

M? Number of categories (map level). The number of retained categories is closely linked to the spatial detail of a polygonal subdivision since the more categories are shown the more polygons will be portrayed. The target scale, the map purpose (e.g. a geology map for a tourist vs. an expert in geology) and the map theme determine the concrete number of categories. Due to its dependence on the specific map that needs to be generalized this constraint represents a typical case where no global rules exist but the user will specify the corresponding threshold values (cf. section 4).

3.2 Topological constraints

T1 Self-intersection (line level). A valid polygonal subdivision of the plane – see Jaakkola (1998) or Frank et al. (1997) for a definition of a polygonal subdivision – must not contain self-intersecting polygon geometries.

T2 Intersection of different polygons (polygon and group level). Intersections of polygon geometries must be avoided since they prohibit the creation of a topologically consistent polygonal subdivision.

3.3 Structural constraints

S1 Shape distortion (polygon level). The distortion of a polygon shape should be minimized, that is, shape characteristics such as angularity or intrinsic micro shapes should change as little as possible.

S2 Absolute position (polygon level). The change of an object's absolute position should be minimized.

S3 Relative configuration (group level). Generalization should maintain as best as possible the direction and distance relations of objects (Yaolin et al. 2001). That is, generalization should preserve not only the positions of polygons, relative to each other, but also characteristics in the spatial distribution of polygons such as alignments, clusters and containments.

S4 Size ratios (group and map level). Size ratios should be preserved in a polygonal subdivision on different levels during generalization, for instance, between polygons of an alignment or a cluster, between polygons of a category and between all categories building a polygonal subdivision.

3.4 Procedural constraints

P1 Illogical results (line, polygon and group level). Generalization should not produce results that are implausible with respect to the spatial (e.g. a phenomenon occurring in compact polygons shown by long and thin polygons) or the semantic component (e.g. impossible neighborhoods of categories) of the represented theme.

P2 Child entity's constraints (polygon, group and map level). Both the hierarchical organization of the spatial levels of polygon generalization and the agent-based approach (Galanda and Weibel 2002a) make so-called parent entities responsible for the generalization of their child entities. For instance, a polygon cluster supervises the independent generalization of its polygons or a polygon controls the generalization of its boundary. This constraint is attached to every parent entity in order to ensure sufficient satisfaction of all those constraints which are delegated to its individual child entities (Ruas 1999, Barrault et al. 2001, Galanda and Weibel 2002a).

P3 Aggregation similarity (group level). This constraint defines the minimum level of semantic similarity required to merge two polygons of different categories. For instance, a polygon of the category 'nursery' is rather aggregated with a polygon of the category 'forest' than with one of the category 'lake'. In other words, the semantic similarity between the categories 'nursery' and 'forest' is closer than the one between the categories 'nursery' and 'lake'. Semantics play a important role in polygon generalization. Since the spatial and semantic components of a polygonal subdivision are intimately linked, and any treatment of one in isolation to the other will have a high risk of misrepresenting the phenomenon (Mark and Csillag 1989).

P4 Equal treatment (all levels). Ensure that similar conflicts are solved in similar ways across the polygonal subdivision.

4 Evaluate constraints

A constraint is always evaluated for an individual object at one of the spatial levels of polygon generalization. The process of evaluating constraints is of great relevance in constraint based approaches to map generalization, since it is responsible for conflict detection, conflict resolution and evaluation of results. In order to fulfill these tasks every constraint needs to be formalized. The formalization of a constraint designates the transformation of a constraint into a formal description that is interpretable by computers. Formalization is based on *goal values* and *measures* (cf. section 2.1).

On a global level, goal values are derived from generalization controls such as map purpose, target scale, limits of perception etc. (Weibel and Dutton 1998) while on a local level the specification of a constraint, that is, the definition of a constraint's goal value is influenced by an object's semantics (Ruas 1999). The specification of goal values directly affects the generalization results, that is, inappropriate goal values can change the intrinsic character of a polygonal subdivision (e.g. too many small polygons are eliminated) or result in an insufficient generalization (e.g. too many details are kept).

The evaluation of a constraint determines the satisfaction of a constraint with respect to a certain object. That is, it compares the result of the measure, calculated on the corresponding object, to the constraint's goal value and derives a degree of satisfaction (=severity) from their discrepancy. However, it is not equally easy to evaluate the satisfaction of different constraints by means of numeric measures. Certain constraints are directly linked to geometric or semantic properties of a polygonal subdivision. Examples include 'M4 Minimal area' or 'M7 Number of categories'. Thus, it is straightforward to establish an appropriate method for their evaluation. Other constraints, such as 'S1 Shape distortion' or 'S3 Relative configuration' are fuzzy and ill-defined (Weibel and Dutton 1998). Hence, they are very difficult to formalize and subsequently to evaluate. More detail on the evaluation of the individual constraints is therefore provided in the following sections.

4.1 Evaluation of metric constraints

As mentioned above metric constraints relate to limits of perception, i.e. minimal dimensions of mapping. Hence, maps, textbooks (e.g. Spiess (2002), SSC (2002)) and guidelines of mapping agencies (e.g. McMaster (1991)) provide sufficient information to specify goal values. Goal values depend mainly on the cartographic representation of the polygonal subdivision and the output media (Weibel and Dutton 1998). Thresholds that are valid for paper maps are listed in Table 1, along with an illustration by solid contours. Note that, goal values are defined in map units in order to ensure independency of the target scale. The measures used for the evaluation of metric constraints are summarized in Table 2.

Evaluate 'M1 Consecutive vertex distance'. The constraint 'M1 Consecutive vertex distance' is evaluated by determining the minimum distance d_{Consec} between any pair of consecutive vertices (v_i, v_{i+1}) along a polygon boundary – cf. Table 2. The constraint's goal value was set to a minimum of 0.1 mm through empirical observation and testing in order to remove exclusively redundant vertices.

Evaluate 'M2 Outline granularity'. An excessive granularity of a polygon outline is defined by the occurrence of imperceptible shapes (micro shapes). Hence, the evaluation of this constraint relies on a preliminary identification of such micro shapes. Similar to the concept proposed by Wang and Müller (1998), it is assumed that every polygon boundary consists of a sequence of external and internal shapes. An external/internal shape is built by a set of subsequent vertices of the polygon boundary that include an internal angle lower/greater than 180° as well as a start and end vertex with an opposite angle (internal angle is greater/lower than 180°). In Figure 2 a polygon is split up into its external shapes (shape 2, 4, 6) and internal shapes (shape 1, 3, 5).



Table 1: Goal values of metric constraints (in map units) for polygon generalization with respect to paper maps. Listed goal values stem from SSC (2002), Spiess (2002) and empirical observation.

According to the mapping guidelines presented in SSC (2002) and Spiess (2002) shapes are regarded as imperceptible if their height falls below 0.4 mm and/or their width falls below 0.6 mm. Here, the height of a shape is measured by calculating the maximum distance of any vertex of a respective shape to the straight line that connects the shape's start and end vertex – cf. the inlet of Figure 2 and Table 2. The width is defined as the distance between the shape's start and end vertex along a straight line – refer again to the inlet of Figure 2 and Table 2.

Evaluate 'M3 Distance between boundary points'. The distance between any points of a polygon geometry is determined by the measure 'detect narrow parts' proposed by Bader and Weibel (1997). This measure identifies narrow parts of a polygon by means of a conforming Delaunay triangulation (Bern and Epstein 1995) built of the points of the polygon boundary. Considering the mapping guidelines of SSC (2002), the goal value of this constraint is set to a value of 0.6 mm.

Evaluate 'M4 Minimal area'. In order to control the satisfaction of the constraint 'M4 Minimal area', a basic area measure is used to calculate the area of a polygon. Considering the most common representation of polygonal subdivisions, that is, solid contours, a minimal area of 4mm² is proposed



Figure 2: A polygon boundary composed of internal and external shapes and the principle of calculating a shape's width and height.

(Malič 1998). The goal value may be varied according to a polygon's semantics, for instance, taking into account whether a polygon belongs to a frequent or rare category or if the polygon's category is considered to be more or less important with respect to map theme and map purpose.

Evaluate 'M5 Respect spatial context'. The evaluation of the constraint 'M5 Respect spatial context' is achieved by intersecting the generalized geometry $geom_G$ of a polygon or a group of polygons with the context geometry $geom_{Con}$, which represents the spatial context of a polygon or group of polygons. In other words, it defines the map space that can be used to establish a generalization of the corresponding polygon or group of polygons without creating a new conflict as a side-effect. Such a geometry can be calculated by using buffering techniques (Boffet and Rocca Serra 2001) while respecting the minimal distance between two polygons – cf. Figure 3. The constraint is satisfied if the generalized geometry $geom_G$ lies completely within the context geometry $geom_{Con}$ – see also Table 2.

Evaluate 'M6 Object separation'. The minimal distance between polygons is defined by the shortest distance d_{Obj} that is found between any pair of polygons (O_i, O_j) of the same group – cf. Table 2. According to (SSC 2002) the distance between polygons should not fall below 0.6 mm.

Evaluate 'M7 Number of categories'. Due to its strong dependency on target scale, map purpose, map theme etc. an optimal number of categories can not be defined *a priori* for arbitrary scales. Thus, its goal value is defined as 'varying' in Table 1.

4.2 Evaluation of topological constraints

Topological constraints belong to the group of offensive constraints, i.e. every data set must adhere to them. A polygonal subdivision can only have two different states concerning topological consistency,



Figure 3: Examples of context geometries for individual polygons (left figure) and a group of polygons (right figure). Context geometries are derived from buffers of the outline of adjacent polygons at the minimal distance between two polygons. The polygons that are generalized are represented by solid contours in dark gray while their adjacent polygons are represented by solid contours in mid-gray . Dashed lines show the derived context geometries.

namely a consistent or an inconsistent one. Thus, a boolean value can describe the satisfaction of such a constraint, i.e. a 'TRUE' denotes topological correctness and a 'FALSE' the occurrence of topological errors. Constraint 'T1 Self-intersection' and 'T2 Intersection of different polygons' are evaluated by geometrical operations checking the generalized object geometries for self-intersections and intersections with other polygons, respectively.

4.3 Evaluation of structural constraints

As explained above, structural constraints control the change of aesthetic properties of a polygonal subdivision during generalization. While in conventional generalization these constraints are often met routinely by cartographers, 'they are often hard to translate into operational terms' (Weibel 1996, p. A.4) for automated generalization. But, it is difficult to interpret the measures used for evaluating structural constraints with respect to the quality of a generalized data set since the structure of a polygonal subdivision is modified during the resolution of metric conflicts. For instance, it would make no sense to reject a solution because of a violated constraint 'S2 Absolute position' if metric constraints are significantly improved in their satisfaction at the same time. Along these lines, Weibel (1996) proposed to initially assume the fulfillment of structural constraints to be the result of all the metric and topological constraints may not only allow comparison of different solutions that equally meet all the other constraints, but also, to maintain the overall visual appearance (Weibel 1996). In other words, the preservation of the overall visual appearance is assumed to result from the satisfaction of all structural constraints.

Evaluate 'S1 Shape distortion'. The amount of shape distortion is measured by comparing shape indices that are calculated for both the original and generalized polygon geometry. The used shape indices are the perimeter-area ratio (FRAGSTATS 1994) and the comparison of a polygon shape to a circular shape of the same area (Peter 2001). Generally, however, shape properties are difficult to describe by numeric values even on the level of an individual polygon (Weibel and Dutton 1998).

Evaluate 'S2 Absolute position'. This constraint is evaluated by calculating the relative area overlap of the original and generalized polygon geometries. Other suitable measures would be the vector and areal displacement (McMaster 1986) or the Hausdorff distance (Hangouët 1995).

Evaluate 'S3 Relative configuration'. The description of relative positions of polygons to each

Constraint	Measure
M1 Consecutive vertex distance	$d_{Consec} = min(\sum_{i=1}^{n} \overline{v_i v_{i+1}})$
M2 Outline granularity	
minimal shape width	$width = min(\sum_{i=1}^{n} \overline{v_1 v_n})$
minimal shape height	$height_{Int} = min(\sum_{i=1}^{n} max(\sum_{j=2}^{n-1} dist(v_j, \overline{v_1 v_n})))$
M3 Distance btw. boundary points	'detect narrow sections' (Bader and Weibel 1997)
M4 Minimum area	'polygon area' (Laser-Scan 1999)
M5 Respect spatial context	$geom_G \cap geom_{Con} = geom_G$
M6 Object separation	$d_{Obj} = min(\sum_{i=1}^{n} \sum_{j=1}^{m} dist(O_i, O_j))$
M7 Number of categories	n_{Categs}

Table 2: Measures for evaluating metric constraints.

other involves the calculation of auxiliary data such as a Delaunay triangulation (Jones et al. 1995, Ruas 1995, Bader and Weibel 1997) or a Minimal Spanning Tree (Regnauld 1998, Bader 2001). The change in relative position results from a quantitative comparison of these geometric structures before and after generalization. Here again, measures such as the vector and areal displacement (McMaster 1986) or the Hausdorff distance (Hangouët 1995) can be applied.

Evaluate 'S4 Size ratios'. Size ratios on both the group and map level are measured through the calculation of relative area values, for instance, the relative area of a category of the total subdivision or the relative area of a polygon of another polygon.

4.4 Evaluation of procedural constraints

Procedural constraints are linked to the generalization process itself rather than to properties of a polygonal subdivision. The evaluation of such a constraint helps to guide the generalization process. That is, it supports decision making, for instance, when examining whether two polygons should be merged or alternatively supervised child entities require generalization.

Evaluate 'P1 Illogical results'. The method used for evaluating the constraint 'P1 Illogical results' depends strongly on the kind of result that would be regarded as illogical and subsequently should be prevented. For instance, an illogical neighborhood such as a lake in the sea is detected by comparing the semantics of adjacent polygons.

Evaluate 'P2 Child entity's constraints'. The satisfaction of this constraint is derived from the average satisfaction of all supervised child entities. The goal value of the constraint 'P2 Child entity's

constraints' defines a minimum level of satisfaction that any child entity should at least reach.

Evaluate 'P3 Aggregation similarity'. The semantic similarity between two polygons is calculated by a measure proposed by Yaolin et al. (2002). This performs similarity evaluation based on the classification schema of the underlying categorical data. A goal value can not be specified a priori since it depends exclusively on the class hierarchy of the generalized data.

Evaluate 'P4 Equal treatment'. This constraint relies on the consideration of two situations being similar. In automated generalization a situation is characterized by a number of factors, such as the constraint that is violated, the objects involved and the severity of the conflict. Since automated analysis of a situation is never as holistic as a cartographer's view and conflict detection is based on given goal values that are interrelated between different constraints there is no guarantee *per se* that similar situations are treated equally.

5 Plans proposed by constraints

Plans are suggested by generalization constraints in order to propose solutions that can help to remedy conflicts that have been detected. Plans can take different forms. On the one hand, if an object does not satisfy a certain constraint this constraint may propose plans that are able to diminish its violation with regard to that object. On the other hand, a constraint may recommend avoiding plans for an object's generalization that are known to degrade its satisfaction (Ruas 1999). In general, plans denote generalization algorithms and respective parameters that could be applied to an object in order to improve the satisfaction of a constraint (Beard 1991, Duchêne et al. 2001). A constraint (e.g. a topological constraint) may also suggest rejecting a generalized data set due to its violation.

Plans are always proposed in consideration of a concrete situation, that is, a constraint is expected to recommend different plans for different situations. This selection of plans also allows generalization strategies to be implemented. For instance, in order to preserve or even emphasize polygons of a specific category, elimination algorithms are avoided as a means for their generalization.

5.1 Plans of metric constraints

Metric constraints are the driving force of map generalization, that is, from their violation originate the need for map generalization and the subsequent transformations related to them accomplish most of the adaptation to the target scale. While the relationship of metric constraints to generalization operations can be established on a general level, the concrete application of a plan always depends on both those algorithms available in a generalization system and the specific situation. Hence, Table 3 provides an overview of metric constraints and attached plans that are defined on the level of generalization operations.

Table 3 shows five different types of relations between metric constraints and generalization operations, namely a *directly positive* ('++'), *indirectly positive* ('+'), *indirectly negative* ('-'), *indefinite* $('\sim')$ and *no* (' ') relation. This distinction is based on the foreseeable influence of a generalization operation on the satisfaction of constraints with respect to a certain object.

Directly positive relation ('++'). A *directly positive relation* between a constraint and a generalization operation occurs if this operation is able to improve the corresponding constraint's satisfaction with respect to an object. For instance, the constraint 'M6 Object separation' may apply a displacement or aggregation algorithm to a group of polygons in order to improve its satisfaction according to this group – cf. Table 3 and Figure 4b&c.

Indirectly positive relation ('+'). An *indirectly positive relation* concerns situations where

	mon	aroup		aroup/polygon		polygon		line	
	map group		group/polygon		polygon		iiiiē		
	Reclassification	Aggregation	Typification	Displacement	Exaggeration	Elimination	Enlargement	Simplification	Smoothing
M1 Consecutive vertex distance	+	~	~		~	+	+	++	++
M2 Outline granularity	+	~	~		~	+	+	++	++
M3 Distance btw. boundary points	+	~	~		++	+	+	+	+
M4 Minimal area	+	++	++		+	++	++	~	~
M5 Respect spatial context	+	++	++	++	++	+	-	~	~
M6 Object separation	+	++	++	++	++	+	-	~	~
M7 Number of categories	++								

Table 3: Metric constraints and plans that are listed on the level of generalization operations. See explanations in the text.

a constraint's satisfaction with respect to a certain object is improved through a generalization operation triggered by another constraint imposed on that object or one of its child entities. Thus, the generalization operation is not directly related to the property improved as a side-effect. For instance, since an elimination operation removes entire polygons from the subdivision, on the one hand, the removed object needs no longer to meet any constraints. On the other hand, satisfaction of constraints (e.g. 'M6 Object separation') may increase for those groups of polygons to which the eliminated polygon belonged – see Figure 4d. Another example is the reclassification operation, i.e. a reduction of the number of categories represented. This semantic transformation usually implies also a reduction in the number of geometric conflicts within a polygonal subdivision (Spiess 1990, Galanda and Weibel 2002b).

Indirectly negative relation ('-'). An *indirectly negative relation* is characterized by the fact that a constraint's satisfaction is deteriorated as a side-effect. For instance, the enlargement of an individual polygon usually implies an aggravation of the satisfaction of the constraint 'M6 Object separation' with respect to the group the enlarged polygon belongs to.

Indefinite relation (' \sim '). Some generalization operations have neither a generally positive nor negative influence on the satisfaction of constraints. Such relations are classified as *indefinite relations*. For instance, there exists no trend as to how the satisfaction of the constraint 'M2 Outline granularity' changes subsequently to an exaggeration operator.

No relation (' '). A relation between a metric constraint and generalization operation is of type *no relation* if the satisfaction of a metric constraint is not influenced by applying the generalization operation. For instance, a displacement operation does not affect the satisfaction of the constraint 'M1 Consecutive vertex distance' since the polygon geometry is translated but its shape remains unchanged.

Figure 4 demonstrates three possible solutions for satisfying a violated constraint 'M6 Object sep-



Figure 4: Improving the satisfaction of the constraint 'M6 Object separation'. a. Original polygons and their buffered outlines at half of the minimal separation distance. Hence, overlaps (white fill color) of buffers mark conflicts. b.&c. Solution of the proximity conflict by a displacement and an aggregation algorithm, respectively (directly positive relation). d. The conflict is solved indirectly through the elimination of the middle polygon which was triggered by a constraint of the respective child entity(indirectly positive relation). The original geometry of modified polygons is displayed by dashed lines.

aration'. As the generalization of an object is controlled by various constraints the satisfaction of this constraint affects other constraints, too. Thus, the best solution is the situation that best compromises the satisfaction of all constraints attached to an object. For instance, the solution achieved by a displacement operation represented in Figure 4b emphasizes the occurrence of three disjoint, independent polygons. This solution is probably the best solution if there is enough space to displace the polygons without creating new proximity conflicts with other polygons and if all the supervised child agents satisfy the constraint 'M4 Minimal area'. The aggregation of disjoint polygons (cf. Figure 4c) is restricted in such a way that the constraint 'P3 Aggregation similarity' must be satisfied, that is, the polygons belong to the same category or to semantically similar categories. The aggregation operation additionally allows satisfaction of the constraint 'M5 Respect spatial context' on the group level and the constraint 'M4 Minimal area' on the level of individual polygons (cf. Table 3). Figure 4d shows the result of an elimination operation. Since the polygon in the middle is removed all conflicts related to this polygon are solved automatically as side-effect (cf. Table 3). As long as only metric constraints are considered this solution is easy to establish and perfect but in fact structural constraints (e.g. 'S3 Relative positions' or 'S4 Size ratios') are violated and so this solution is considered as suboptimal.

While metric constraints trigger semantic and geometric transformations of data sets structural and topological constraints control mainly the acceptance of these changes. Plans proposed by these constraints and procedural constraints are discussed in the following section.

5.2 Plans of other constraints

Topological constraints. Since a valid state of a polygonal subdivision must not include any selfintersection or intersections between polygon boundaries, every violation of a topological constraint leads stringently to a rejection of a generalized data set. Generalization algorithms may observe such constraints explicitly in order to ensure topological correct solutions. For instance, Edwardes et al. (1998), de Berg et al. (1998) and Saalfeld (1999) proposed such algorithms for the simplification of polygon boundaries. If non-topologically aware generalization algorithms are used, however, an *a posteriori* topological check may establish whether the topological correctness has been maintained.

Structural constraints. Structural constraints belonging to the group of defensive constraints, rather control the maintenance of the spatial and semantic structure than initiate modifications of a polygonal subdivision. It is recommended that their violation stays within certain limits to avoid visually unbalanced results and unnatural size relations (Weibel and Dutton 1998). The identification of meaningful relations between the violation of structural constraints and the necessity of generalization, that is, to meet other constraints, is difficult. Generally, every structural constraint is able to reject a solution provided by the generalization process if its violations exceeds a predefined limit. In practice, structural constraints are mainly used to compare different solutions that equally meet other, mainly metric, constraints (cf. section 4.3).

Procedural constraints. Procedural constraints do not have the general characteristic of what type of plans are to be proposed if they are violated. While the constraint 'P4 Equal treatment', that designates a basic principle of generalization, does not suggest any plans the constraint 'P1 Illogical results' is – as topological constraints – able to reject a generalized data set if it is violated. As mentioned above, the constraint 'P3 Aggregation similarity' evaluates if the aggregation of two polygons put forward by another constraint is feasible. That is, if the calculated semantic similarity exceeds the required goal value this plan is triggered, otherwise it is refused. The constraint 'P2 Child entity's satisfaction' is essential in agent-based generalization as it triggers and controls the generalization of supervised child entity that does not meet all its constraints. For instance, the generalization of polygons is always controlled by a parent entity (cf. (Galanda and Weibel 2002a)). That means, a parent entity is responsible to satisfy not only its own constraints but also its child entities.

6 Importance of constraints

In practice, constraint based generalization is characterized by the fact that every cartographic object has to meet several constraints. Hence, solutions must compromise between different, and most likely even contradicting constraints. For accomplishing such a compromise importance values on constraints are needed that define if a constraint must be met or could be relaxed in comparison to other constraints (Beard 1991, Ruas 1999).

Generic importance. Topological constraints generally hold a higher importance than metric constraints. They must be fulfilled in order to obtain a sound polygonal subdivision whilst metric constraints need not to be satisfied *per se*, that is, they can be relaxed if necessary. As explained above, it is difficult to define meaningful relations between the violation of structural constraints and the quality of a generalized data set. Additionally, some concepts such as shape are fuzzy and ill-defined (Weibel and Dutton 1998). Thus, structural constraints are assigned the lowest importance although these constraints are from a cartographic point of view highly significant for a fully satisfactory generalization (Weibel 1996). The procedural constraints 'P3 Aggregation similarity' and 'P4 Equal treatment' are not looked at for defining importance of constraints since they are linked to the generalization process itself rather than to individual objects. While the constraint 'P2

Child entity's constraints' should receive a similar importance to metric constraints, the importance of constraint 'P1 Illogical results' is not taken into account for the generic importance of constraints discussed below since its importance depends – as discussed previously – too much on the kind of illogical result that should be prevented.

Importance is only relevant amongst constraints referring to the same spatial level of polygon generalization, that is, importance of constraints must be defined separately for the line, polygon, group and map level. Table 4 lists the generic importance of individual constraints that were derived from a priori knowledge, textbooks and empirical testing at these levels of polygon generalization.

Spatial level	Constraints (ordered from highest to lowest importance)
Line level	T1 Self-intersection
	M2 Outline granularity
	M1 Consecutive vertex distance
Polygon level	T2 Intersection of different polygons
	M4 Minimal area
	P2 Child entity's constraints
	M3 Distance between boundary points
	M5 Respect spatial context
	S1 Shape distortion, S2 Absolute position
Group level	T2 Intersection of different polygons
	P2 Child entity's constraints
	M5 Respect spatial context M6 Object separation
	S3 Relative configuration, S4 Size ratios
Map level	M7 Number of categories, P2 Child entity's constraints
	S4 Size ratios

Table 4: Importance of constraints on the line, polygon, group and map level, respectively.

Importance on the line level. The constraint 'M1 Consecutive vertex distance' aims only at removing duplicated points, that is, it is not a prerequisite of proper generalization. Hence, the constraint 'M2 Outline granularity' receives a higher importance than 'M1 Consecutive vertex distance', while the constraint 'T1 Self-intersection' holds the highest importance at the line level.

Importance on the polygon level. On the polygon level the constraint 'M4 Minimal area' takes precedence over all the other constraints with the exception of the topological constraint 'T2 Intersection of different polygons'. As long this metric constraint is unsatisfied it makes no sense to consider other constraints since if a polygon is too small to be perceivable other constraints are only of secondary interest. Empirical testing emphasized that the constraints at the line level controlled by 'P2 Child entity's constraints' commonly affect the entire polygon, while the constraint 'M3 Distance between boundary points' has, rather, a local impact. Thus, the procedural constraint obtains higher importance than the metric constraint. The constraint 'M5 Respect spatial context' receives the second lowest importance since proximity conflicts are better solved at the group level. Finally, the lowest importance is assigned to the structural constraints since their ranking Absolute position'.

seems to be reasonable only in consideration of user's needs and preferences.

Importance on the group level. At the group level, topological constraints are again of the highest importance, likewise structural constraints again obtain the lowest importance. A preliminary generalization of the child entities, i.e. polygons, attached to a group level is a prerequisite for a meaningful evaluation of the metric constraints 'M5 Respect spatial context' and 'M6 Object separation'. Hence, both constraints are assigned a lower importance than the constraint 'P2 Child entity's constraints'. While the constraint 'M6 Object separation' concerns distances between objects of the same group the constraint, 'M5 Respect spatial context' refers to distances between the entire group and the spatial neighborhood. Thus, they are assumed to be interrelated and receive equal importance in the generalization process.

Importance on the map level. On the map level the constraints 'M7 Number of categories' and 'P2 Child entity's constraints' hold the highest priority. While the metric constraint controls the semantic generalization, that is, a reduction of the number of shown categories, the procedural constraint initiates adaptations of the polygons' geometries to the target scale if need be. To achieve an appropriate generalization both processes are equally important. The lowest importance is assigned to the constraint 'S4 Size ratios'.

These generic importance values of constraints for polygon generalization must be adapted specifically to the conducted generalization task, which is characterized by the map purpose, the given kind of categorical data, the users' needs and preferences etc. (Ruas and Plazanet 1996).

7 Prioritization of constraints

Priorities of constraints allow procedural knowledge to be considered in the generalization process according to which constraint should be satisfied prior to others, that is, an 'optimal' sequence of constraint satisfaction can be indicated (Regnauld 2001). The priorities discussed below and summarized in Table 5 are derived from empirical knowledge and testing. As with, the importance of constraints, priorities are only relevant among constraints that refer to the same level of polygon generalization. Additionally, the only constraints need to be considered are those that propose plans that result in transformations of the data set. Note that, whenever a constraint, such as a topological constraint, demands the rejection of a solution a backtrack to the previous state is imperatively performed without taking into account any other plans.

Priorities at the line level. On the line level the constraint 'M1 Consecutive vertex distance' receives the highest priority. Its satisfaction helps to speed up the resolution of subsequent conflicts such the constraint 'M2 Outline granularity', by removing redundant vertices from polygon boundaries.

Priorities at the polygon level. The constraint 'M4 Minimal area' receives the highest priority on the polygon level as the satisfaction of this constraint may go along with the satisfaction of the constraints 'M3 Distance between boundary points' and 'P2 Child entity's constraints'. For instance, the enlargement of a polygon may also solve conflicts related to supervised line (child) entities such as an excessive granularity of polygon outlines. Since the procedural constraint controls the generalization of the polygon boundary and the removal of redundant points, respectively, it receives a higher priority than the constraint 'M3 Distance between boundary points'. The resolution of this metric constraint can be significantly speeded up by a reduced number of vertices along the polygon boundary initiated by 'P2 Child entity's constraints'. As an evaluation of the constraint 'M5 Respect spatial context' relies on a completed generalization of the polygon geometry it obtains the lowest priority for the polygon level.

Spatial level	Constraints (ordered from highest to lowest priority)
Line level	M2 Outline granularity
	M1 Consecutive vertex distance
Polygon level	M4 Minimal area
	P2 Child entity's constraints
	M3 Distance between boundary points
	M5 Respect spatial context
Group level	P2 Child entity's constraints
	M5 Respect spatial context, M6 Object separation
Map level	M7 Number of categories
	P2 Child entity's constraints

Table 5: Priorities of constraints on the line, polygon, group and map level, respectively.

Priorities at the group level. The satisfaction of the constraints attached to supervised child agents may also affect distances between objects. Hence, the constraints 'M5 Respect spatial context' and 'M6 Object separation' receive lower priority than the constraint 'P2 Child entity's constraints'. As already discussed above, both metric constraints are interrelated and thus obtain equal priority.

Priorities at the map level. At the map level the constraint 'M7 Number of categories' receives the highest priority since a reduction of the number of classes represented in the target map also implies a reduction in number of polygons and possible conflicts. Thus, child entities are identified and generalized autonomously, that is, the constraint 'P2 Child entity's constraints' holds a lower priority than the metric constraint 'M7 Number of categories'.

The priorities and severities of the constraints attached to a certain object help to detect the best plan for starting the generalization process of the corresponding object. The identification and subsequent triggering of the best plan allows the iterative generalization process to be sped up since it is hoped that a perfect solution (state), that is, a complete satisfaction of all constraints, is reached earlier using this heuristic. For a detailed discussion on the principle of decision making in the AGENT engine refer to Regnauld (2001), Barrault et al. (2001) and Duchêne and Regnauld (2002).

8 Conclusions and outlook

The paper suggested a preliminary set of constraints that intends to cover the basic requirements of polygon generalization. In continuation of previous research (Weibel 1996, Peter and Weibel 1999) this set was raised and individual constraints discussed at a conceptual level. The novelty of this paper lay in the enhancement of the individual constraints by a method for their evaluation based on goal values and measures, by a list of plans on the level of a generalization operations, a priority value and an importance value as a basis of compromising between several competing constraints. In doing so, the following points stood out:

• The evaluation and embedding of structural constraints in the generalization process requires additional research effort since concepts such as 'shape' or 'visual balance' are not established

sufficiently for providing suitable methods for their evaluation and/or for their direct linkage to the quality of generalization.

- Plans need further specification, that is, the plans listed on the level of generalization operations must be marked down to algorithms and parameters in consideration of available algorithms, generalization controls (e.g. target scale, map purpose, user's needs etc.) and the situation that has to be solved.
- The generic importance and priority values assigned to constraints at the spatial levels of polygon generalization provide a foundation for any implementation of these constraints. Additional testing may allow insight to be gained answering whether the magnitude of the covered scale change or the specific type of polygonal subdivision (e.g. geology, land cover) influences these generic importance and priority rankings of constraints.
- Although this set of constraints was set up with respect to our general research goal, i.e. the automation of polygon generalization by means of a MAS, it seems to be a valuable starting point for the implementation of any system for automated polygon generalization.

As mentioned above this preliminary set of constraints shall be integrated into the agent-based framework for polygon generalization of Galanda and Weibel (2002a). As the research reported here is a continuation of the AGENT project the implementation of this framework is based on the generalization engine developed during the AGENT project (Barrault et al. 2001). Consequently, the geographic information system LAMPS2 of Laser-Scan Ltd., that contains the AGENT prototype, serves as the development platform. In doing the implementation, the proposed generalization (Galanda and Weibel 2002b) and the AGENT engine into a coherent, automated generalization process. The prototype will allow empirically testing. The results of the planned experiments intend to achieve both the evaluation and iterative enhancement of the proposed constraints and their evaluation methods. The middle-term goal is, then, to arrive at conclusions on the potential of the proposed constraints not only in isolation from each other but also in interaction with each other for automated polygon generalization. Beyond this, the work reported here is a prerequisite for achieving an evaluation of the capacities of the proposed framework in particular and the agent-based approach (Ruas 1999, Barrault et al. 2001) in general.

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