

# Local length ratio as a measure of critical points detection for line simplification

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## Structure:

Introduction

Literature review

Aims of the study

The measure of local length ratio (LLR)

Comparison with relevant research

Thapa line

Marino's geomorphological lines

Application of LLR measure for assessing line simplification

Peristera Island coastline

Concluding remarks

### Attneave (1954)

Critical points: “It is clear that subjects show a great deal of agreement in their abstractions of points best representing the shape, and most of these points are taken from regions where the contour is most different from a straight line”

Critical points are able to describe and characterize the shape of the object, since they transmit more information than the others

### Marino (1979)

Underlines the impact of critical points in line generalization because these points are recognizable both by cartographers and non-cartographers at the same degree

### Freeman (1978)

Includes in his definition of critical points the points along a digital line that are:

- curvature maxima
- curvature minima
- end points
- points of intersection

Critical points have attracted interest in research

- Cartography:

Marino 1979, White 1985, Thapa 1987,1988a, 1988b, 1989, McMaster 1987, Jenks 1989, Visvalingam and Whyatt 1990, Dutton 1999

(Related to generalization)

- Computer science (computer vision, pattern recognition, or signal processing):

Pavlidis and Horovitch 1974, Iñesta et al 1998, Cronin 1999, Neumann and Teisseron 2002

(Related to line approximation, curve segmentation, or feature detection)

Li (1995)

Provides a comprehensive review of critical points detection algorithms that can be applied on digital cartographic lines describing their advantages and disadvantages

The majority of the existing algorithms of critical points detection are mainly focused on the estimation of the curvature of the contour of a graphical object at any location

Antoine et al 1997

Introduced algorithms for detecting critical points based on wavelets

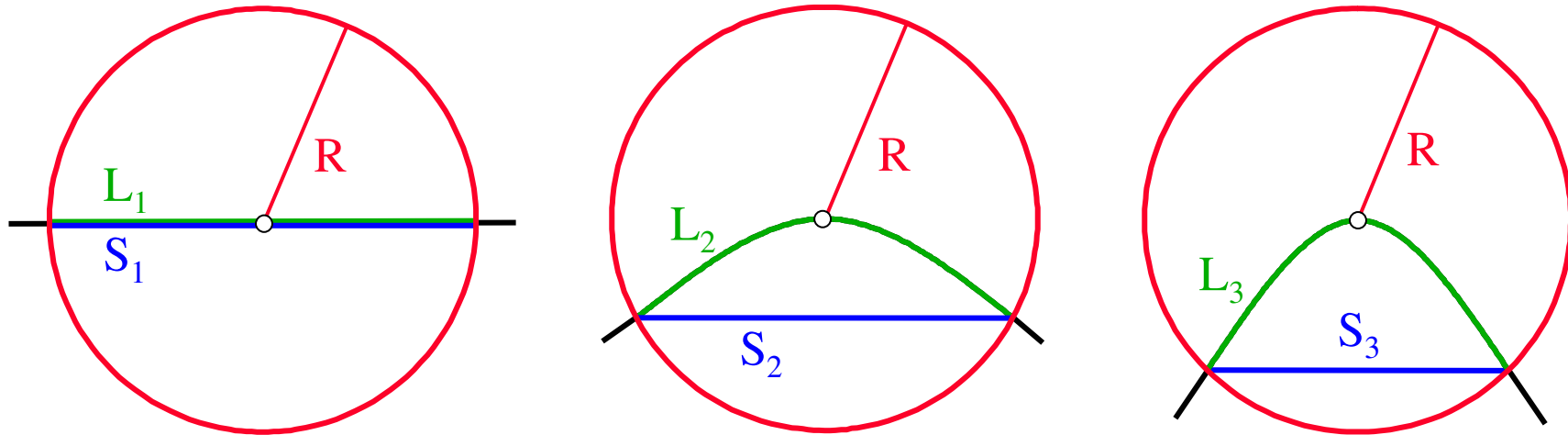
Cornic 1997

Introduced adaptive algorithms that require no input parameters

Considering a cartographic line, that is usually expressed digitally in vector structure (discrete representation), there is no way of applying directly the mathematical definition of curvature

Cornic 1997

- To avoid the drawbacks of the discrete representation some studies smooth the original curve by a Gaussian filter before computing extreme curvature points
- Two main problems: a rather small Gaussian filter width may lead to insignificant detections, whereas a large width may exclude certain critical points from detection

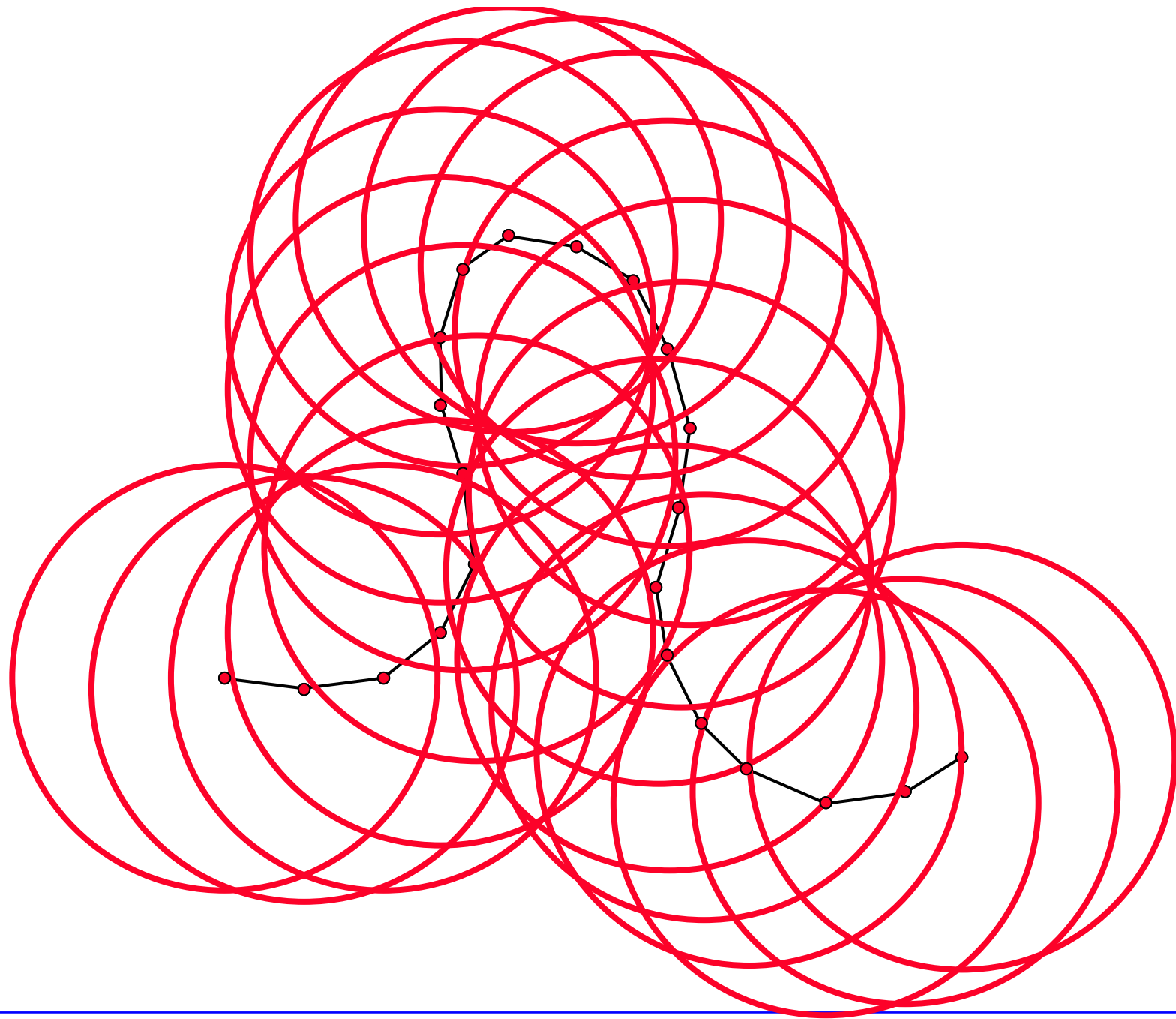


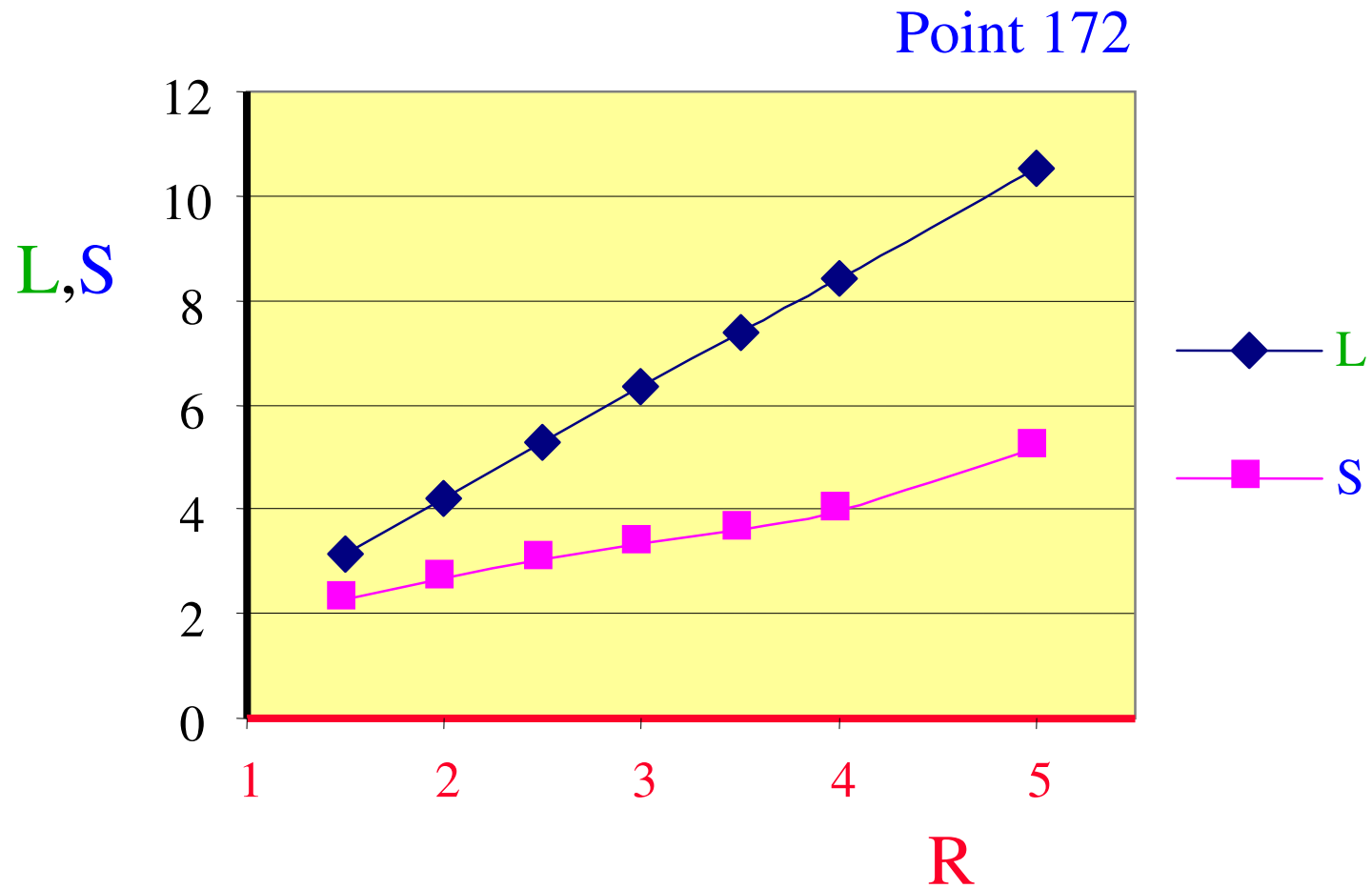
$$L_1 < L_2 < L_3 \quad S_1 > S_2 > S_3$$

Local length ratio:

$$\text{LLR} = \frac{L}{S}$$

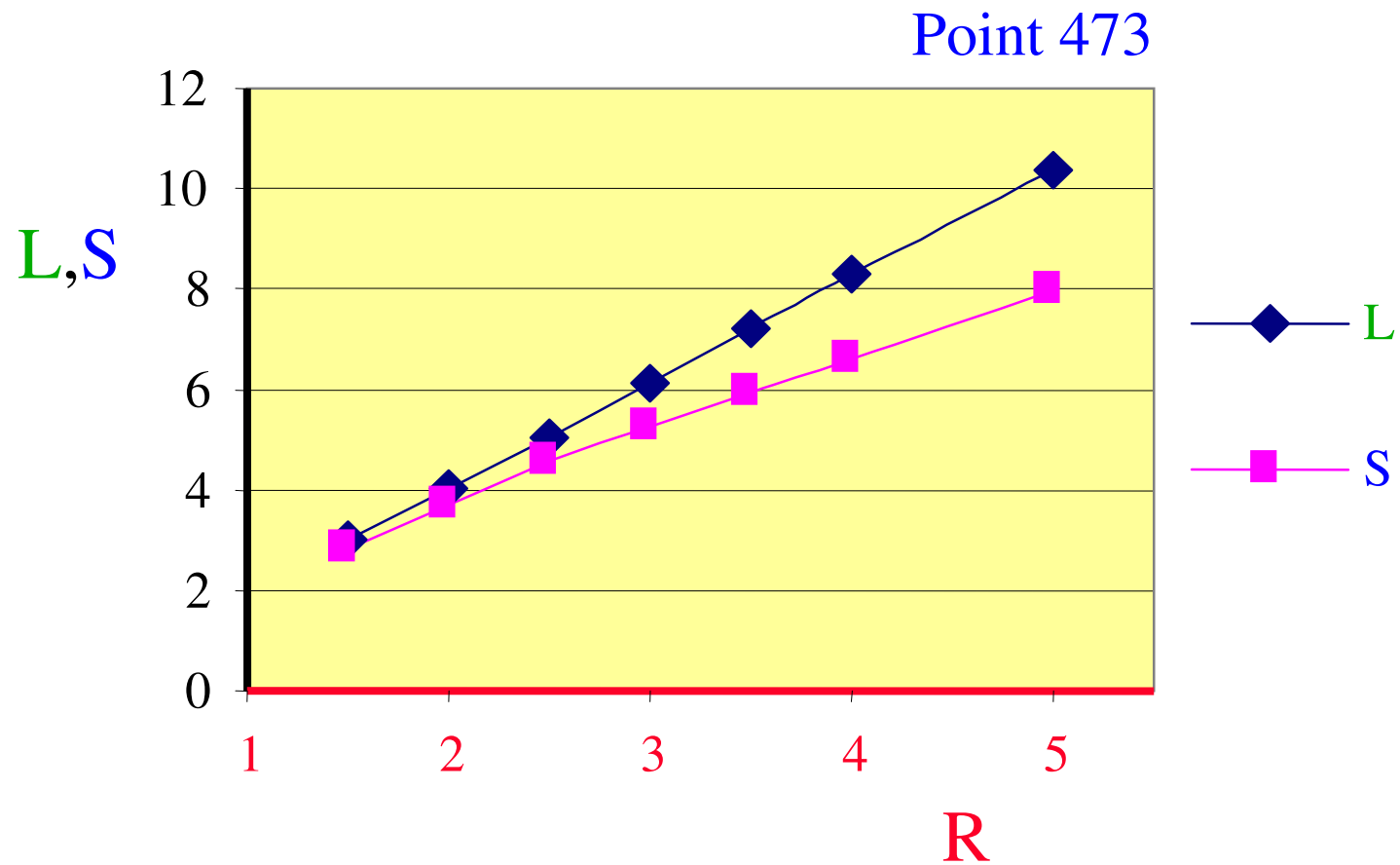
$$\text{LLR} > 1$$



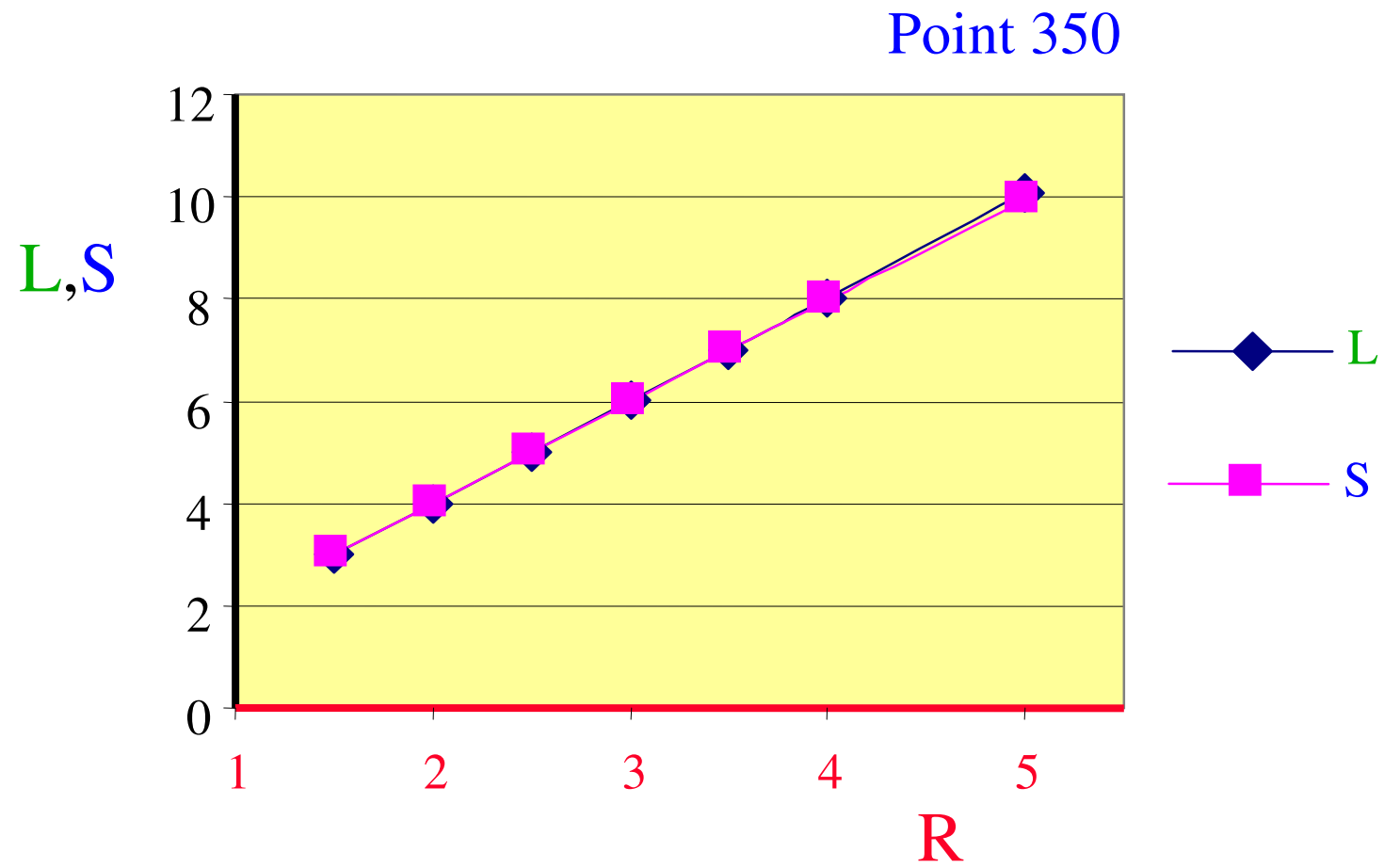


Point 172: a location of sharp slope variation.

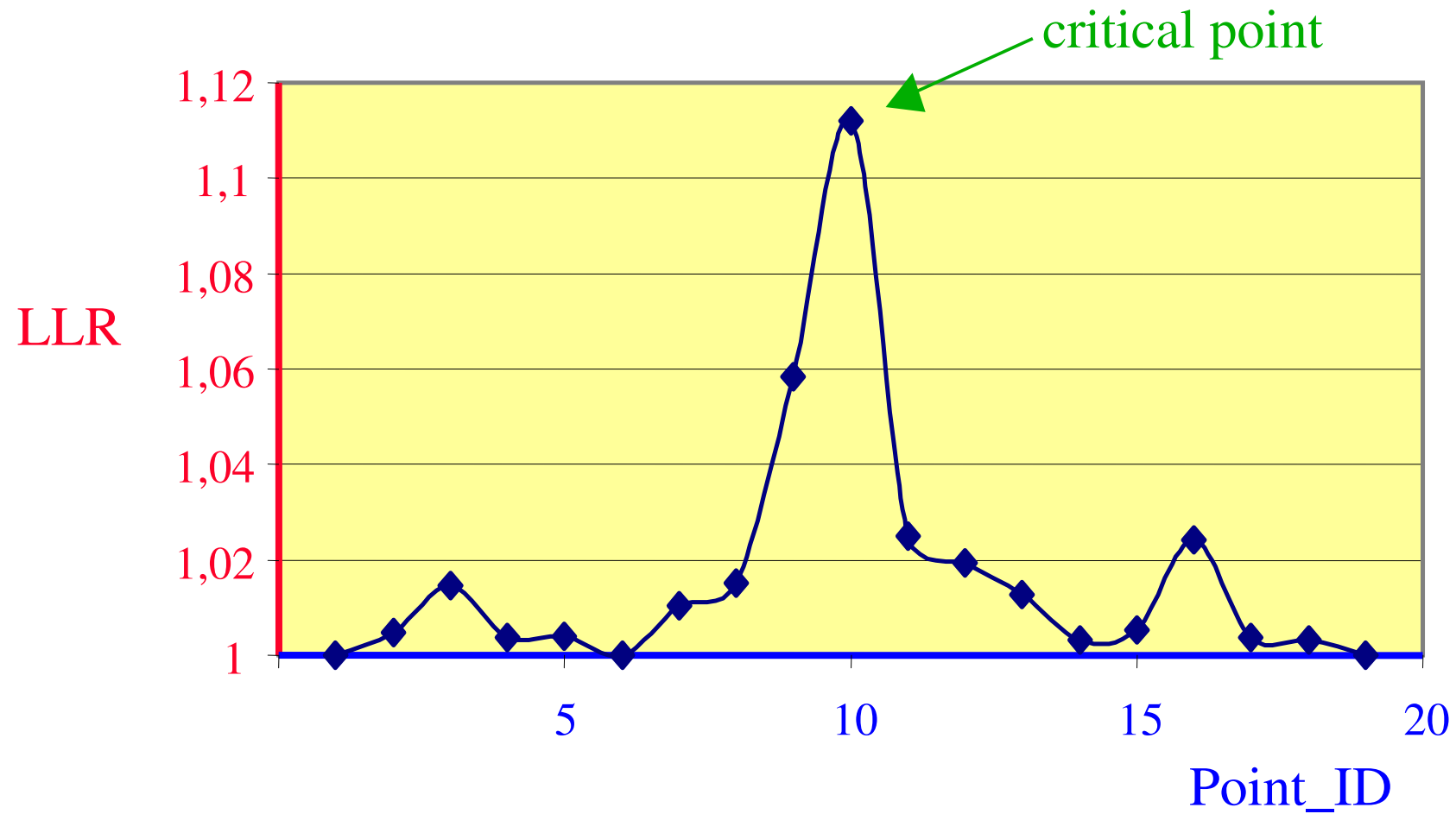


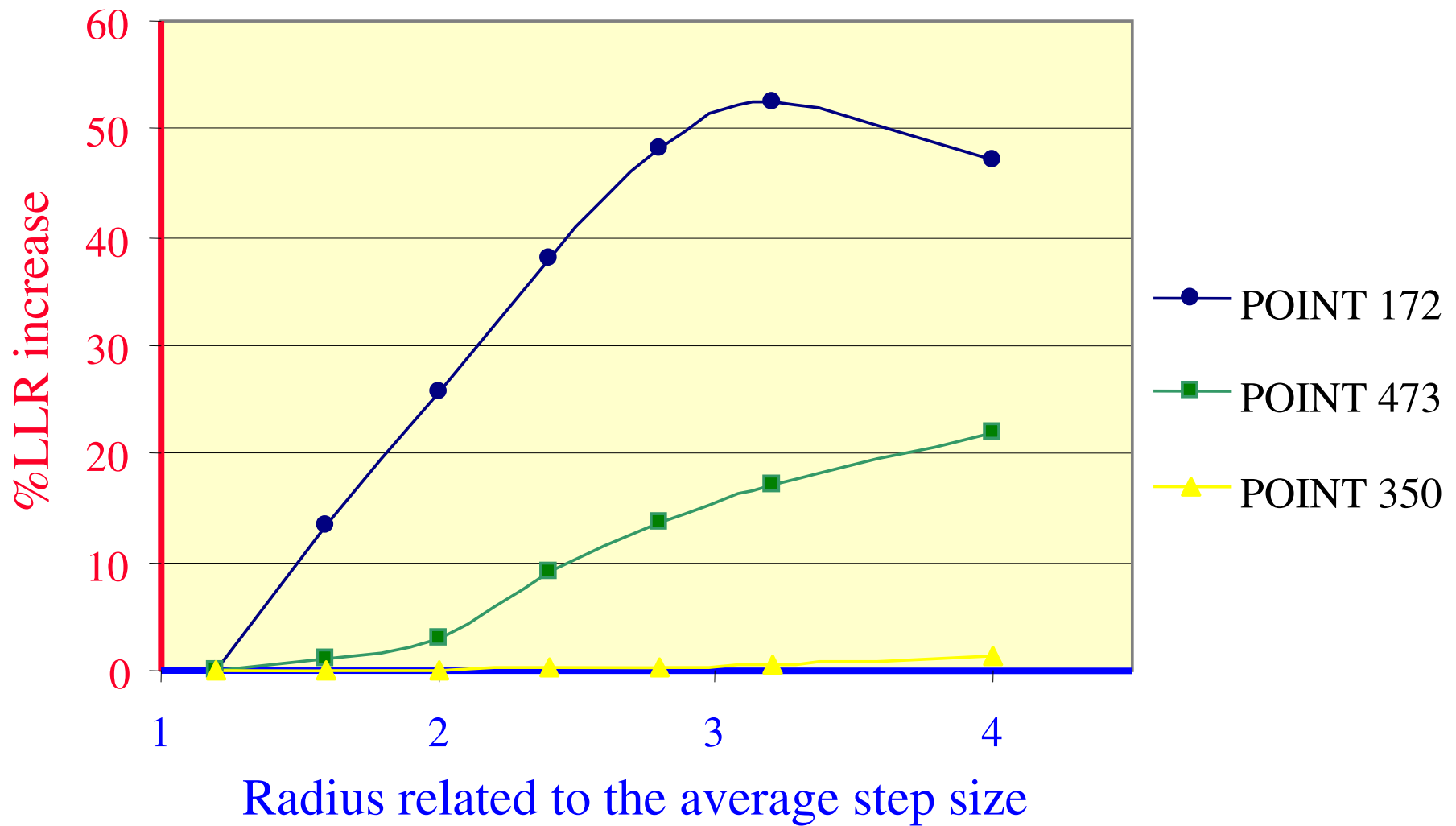


Point 473: a location of a curve peak



Point 350: a location of straight section beginning





- Group A:

LLR values ranged between 1.04 and 1.15 correspond to locations of smoothed slopes (up to 120°) that form fluctuations with height vs. basis ratio of 1:3.5 or smaller

- Group B:

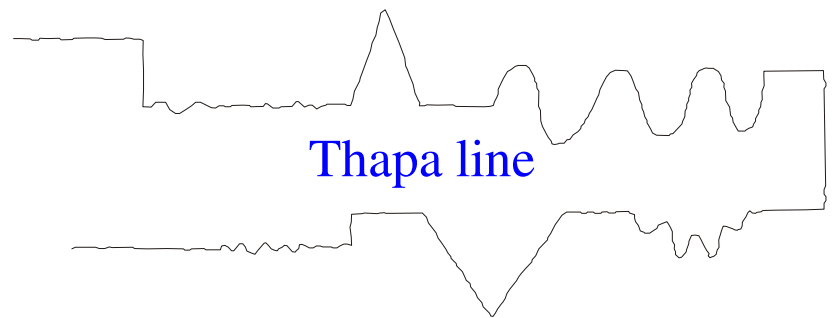
LLR values ranged between 1.15 and 1.30 correspond to locations of sharp slope changes (90°-120°) that form “heavy” fluctuations with height vs. basis ratio between 1:3.5 to 1:2.5

- Group C:

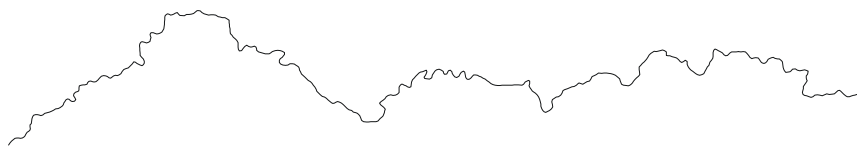
LLR values greater than 1.30 correspond to locations of peaks with slopes less than 90° that form peaks with height vs. basis ratio higher than 1:2.5



Cape Argo



Thapa line

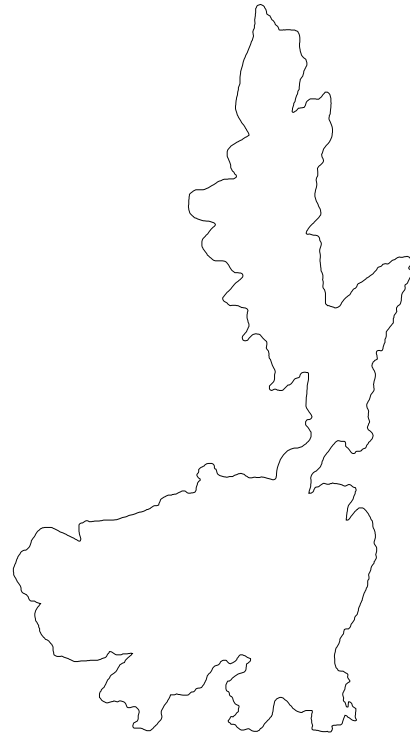


Mancos River



Shenandoah River

Line	Average Step Size (du)	Tol.	Radius (du)	Known C-P	Detected C-P	Common C-P (%)
Thapa line	2.50	1.04	5	45	41	91
Mancos River	1.18	1.04	2,5	40	38	95
Shenandoah River	1.21	1.04	2,5	53	53	100
Cape Argo Coastline	1.22	1.04	2,5	53	49	92



Peristera Island coastline



C-Ps connected with  
line segments



Level	Algorithm	Tolerance (drawing units)	Retained Vertices	Retained Vertices (%)
1	D-P	0.2	2131	88
	Bendsimplify	19.3	2131	
2	D-P	0.5	1733	71
	Bendsimplify	41.5	1728	
3	D-P	1	1333	55
	Bendsimplify	70	1333	
4	D-P	2	963	40
	Bendsimplify	124	960	
5	D-P	5	542	22
	Bendsimplify	240	546	
6	D-P	10	351	14
	Bendsimplify	325	353	

Level	Douglas and Peucker algorithm		Bendsimplify algorithm	
	Remaining C-P	Remaining C-P (%)	Remaining C-P	Remaining C-P (%)
1	153	100	153	100
2	153	100	148	97
3	153	100	131	86
4	152	99	112	73
5	151	99	76	50
6	147	96	56	37

	Douglas and Peucker algorithm						Bendsimplify algorithm					
Level	1	2	3	4	5	6	1	2	3	4	5	6
Group A	103	103	103	102	102	98	103	98	86	74	52	39
Group B	31	31	31	31	31	31	31	31	29	26	19	13
Group C	19	19	19	19	18	18	19	19	16	12	5	4

## Concluding remarks

The proposed measure of critical points detection is:

- simple in its conception
- easily implemented

Similar results with:

- the method proposed by Thapa (1988b)
- perceptually detected critical points by cartographers and non-cartographers (Marino 1979)

LLR also found useful to assess the two line simplification algorithms:

- it came out that the Douglas and Peucker algorithm retains parts of the line that are of high complexity by preserving almost all the critical points at all levels of simplification
- Bendsimplify algorithm retains a minimum of the line complexity but the maximum of its curvature

LLR values, associated with each vertex of the line and saved in a table, could be transformed into useful information for multi-scale representations of a line