

## **How to merge optimization and agent-based techniques in a single generalization model?**

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### **1. Introduction**

Many works in generalization automation concern the conception of generalization models. The role of generalization models is to get a complete framework to perform the complete generalization of a geographic dataset. In most of them, generalization is seen as a constraint satisfaction problem. Constraints are made explicit following (Beard, 1991), and are a translation of the final map requirements. Some constraints concern the legibility of the objects (for example, objects must not be too closed), and force their geometry to change (too closed objects are displaced), while other constraints force to preserve some characteristics of the objects (an object should preserve its position and its shape). Generalization models aim to find a way to manage the satisfaction of these change and preservation constraints.

In this paper, we focus especially on two families of generalization models: Optimization-based models, illustrated by the works of Sester (2005), Harrie & Sarjakoski (2002), Højholt (2000), Bader (2001), Burghardt & Meier (1997), and Agent-based models of Duchêne (2004), Ruas (1999), and the AGENT project (Barrault et al., 2001). An important difference between optimization and agent-based models comes from the way the constraints are considered. In the optimization models, the constraints are satisfied altogether in one step, using a global resolution method to find a compromise between them: all the constraints are “elastic” and a balance between them is found. In the agent-based models, constraints are satisfied step by step, by triggering an algorithm to solve an identified cartographic conflict. The constraints are satisfied depending on an importance value. The result is not a compromise between the constraints: the most important constraints are satisfied totally while others, less important, are relaxed.

These two families of models have provided very good improvements and are now used in several map series production lines as presented in (Lemarié, 2003; Lecordix, 2005). The purpose of this article is to show that these models have different application fields and combining it would allow to improve the automatic generalization process. We show that optimization-based models are much more adapted to compute “continuous transformations”, such as deformations, while agent-based models are adapted to “discrete transformation”. We introduce the notion of “malleable” and “rigid” objects.

In the first part of this article, we give a description of some discrete and continuous operations through the analysis of a manually generalized map example. Then, we give the principles of the optimization and agent-based generalization models and show why the

application field of optimization and agent-based models are respectively continuous and discrete transformations. We present the benefits to merge these techniques.

In a second part, we propose some elements to progress toward a single generalization model. We propose an agent-based model to compute continuous transformations. This model would allow giving to the objects a malleable behaviour and control their deformation. The principle of the model is to consider the points composing the geometry of the objects as agents to allow them to satisfy a set of elastic constraints. We finally give some results obtained from the implementation of the proposed method.

## 2. Discrete and continuous operation in automated map generalization

### 2.1 An example

In this part, we study an example of a manually generalized map in order to illustrate the notions of ‘discrete’ and ‘continuous’ operations. Figure 1 presents two maps of the same area at different scales (1:50k and 1:100k). Red circles and arrows show homologue situations in both maps. We describe the transformation performed to get the generalized result.

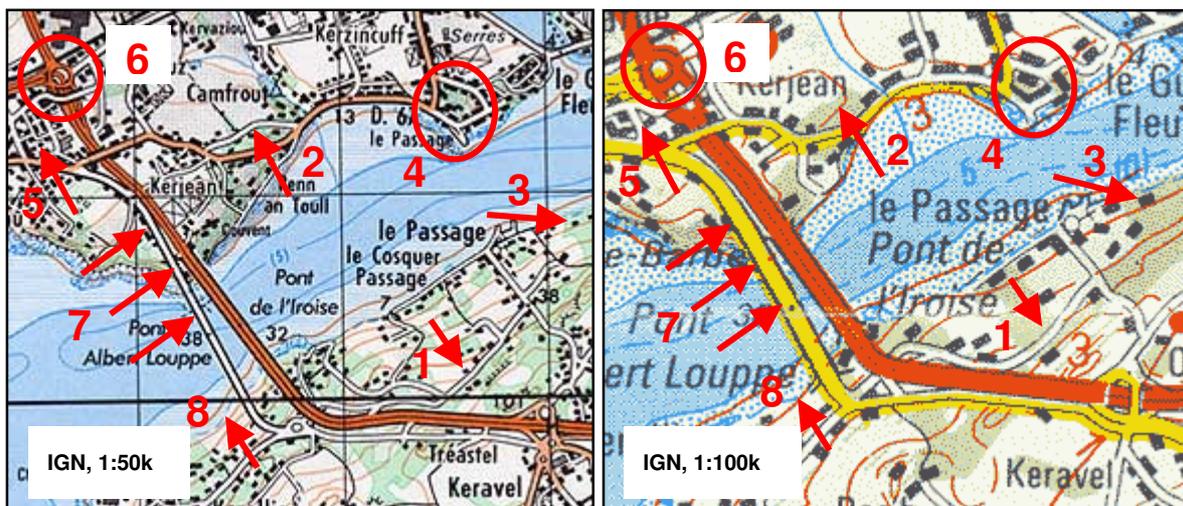


Figure 1. Examples of discrete and continuous operations

- 1 Building deletion
- 2 Road deletion
- 3 Building dilatation: a too little building is enlarged to become visible enough.
- 4 Building group typification: the group of buildings in the urban block is modified. The density, the repartition and the shape of the buildings in the result representation are preserved.
- 5 Road network typification: some white roads are deleted. In the resulting representation, the roads are still orthogonal.
- 6 Interchange simplification and dilation: some access roads of the interchange have been

deleted. The roundabout is enlarged; the structure of the interchange is caricatured.

**7 Road part displacement and deformation:** a part of the white/yellow road is displaced to avoid an overlapping with the highway. This displacement is propagated to the network to preserve the straight shape of both roads.

**8 Contour lines smoothing:** details of the relief are erased by applying a light smoothing operation to the contour lines.

Among the 8 described operations, some changes are much bigger than others. The consequence of the transformations 1, 2, 3, 4, 5 and 6 is a big change of the representation. These transformations are a break with the initial data. Some characteristics, some structure of the initial data are erased or underlined. These operations are not really a transformation: a new representation of the objects seems to be drawn.

Some others transformations such as 7 and 8 are rather smooth changes. These changes are light deformations to erase some details (8) or preserve others (the propagation in 7 allows preserving the straight shape of the road). The result is still close to the initial state of the object: It seems still possible and easy to link the initial and the final states of the objects.

The difference between this two kinds of transformations has been noticed in some map generalisation works (Harrie and Sarjakoski, 2002; Sester, 2005). Operation of the first family are “discrete transformations”, others are “continuous transformation”. (Van Kreveld, 2001) asserts that a generalisation operation can be considered either in a continuous frame, either in a discrete one. For example, the displacement of an object can be seen as a smooth and continuous transformation, or as a discrete one.

During a generalization process, the choice of one of these transformation types mainly depends on two important factors:

- **The scale change amplitude:** for little scale changes, only deformation could be enough to get a satisfying result. When the scale change increases, discrete transformations must be applied, as illustrated in the smooth zooming progress of Van Kreveld (2001).
- **The type of the objects:** some objects have properties, which force them to be subjected to either continuous or discrete transformations. Harrie and Sarjakoski, (2002) underlines the fact that objects such as buildings are much more “rigid” than the other like road “plastic” - or “malleable”. Roads are deformed because one of their properties is to preserve the topology of the network (connection between sections).

However these two factors give only a general trend concerning the usage of either continuous or discrete transformations. In some cases, even when a scale change is little, discrete transformations have to be performed (especially objects eliminations). Furthermore, an object is not either rigid, either malleable, but it can need to be both. Many objects can be subjected to discrete and continuous operations. For example, operations on a road section could be discrete (bend removal, bend succession typification, or deletion) or continuous (deformation, propagation of a displacement to preserve the topology of the network). The fact to be malleable or rigid does not seem to be a static property of an object, it is rather a behaviour it can have depending on the stage of the generalisation process.

Furthermore, in many generalization cases continuous transformations appears to be useful to manage side effects of discrete transformations. For example, the road deformation (transformation 7 of figure 1) is a propagation of the displacement of a part of the road (to avoid the overlapping). It seems to be a side effect of a discrete transformation computed in order to preserve some characteristics of the network (the straight shape of the road section)

## **2.2 Discrete and continuous operations in the optimization and agent-based models**

In the previous section, we have presented the continuous and discrete transformations. In this section, we give a description of the optimization and agent-based models. We show that optimization based techniques are much more adapted for continuous transformations, while agent-based are adapted for a discrete ones. Then we present some problems to tackle to progress toward a merged generalization model.

### **2.2.1 Optimization-based models**

Works on optimisation-based techniques for map generalization uses various concepts such as snakes (Burghardt and Meier, 1997), elastic beams (Bader, 2001), flexible triangles (Højholt, 2000), least square adjustment and conjugate gradients method (Harrie and Sarjakoski, 2002), (Sester, 2005). The purpose of these methods is to determine an adequate displacement of the points composing the geometries of the objects in order to reach a balance position between change and preservation constraints. Constraints are translated into an equations system on the coordinates of the points. This system is globally solved to determine the displacements of the points, using a matrix inversion based method (finite element method, least square adjustment).

Because these models lie on the search of a balance between preservation and change constraints, the shape characteristics of the objects are well preserved; it is often easy to make a link between the initial and the final representation. Consequently, these models are rather adapted for continuous transformations.

### **2.2.2 Agent-based models**

In the agent-based models presented in (Ruas, 1999), (Barrault et al., 2001) and (Duchêne, 2004), the generalization process is seen as a sequence of treatments (dilatation, deletion, displacement, squaring, shape transformation...). Each treatment allows solving cartographic conflicts progressively. In these models, geographic objects are considered as agents: they have a goal and try to reach it autonomously. Their goal is to satisfy their cartographic constraints. To achieve its goal, an agent is able to measure and analyze the state of its constraints, and then to choose and trigger an adequate algorithm in order to improve its general state. Each agent tries transformations until it has reached a satisfied state (Ruas and Plazanet, 1996). This approach is based on the works of (Brassel and Weibel, 1988), (McMaster and Shea, 1988) and (Shea and McMaster, 1989). These works underline the necessity to analyse the data before their generalization. This analysis allows determining which treatment must be applied to the right object(s), at a good stage of the process. The analysis of the data is included in the generalization process.

In these models, each treatment is validated only if it has allowed improving significantly the considered cartographic conflict. The resulting generalization process is a sequence of discrete changes. Consequently, these models are rather adapted for discrete transformations.

### **2.2.3 Toward a merged model**

As illustrated in the example in part 2.1, a complete generalization model should be able to manage both types of transformations.

How could discrete transformations be managed in optimization-based models? Because it uses a global resolution method based on matrix inversion, the inclusion of discrete transformations in optimization-based models seems hard to do. Usually, the frameworks using these techniques propose to compute discrete transformations such as deletions in a pre-processing stage (Harrie and Sarjakoski, 2002; Brenner and Sester 2005). Discrete transformations are applied first, and then continuous transformations.

We aim rather to propose a way to include continuous transformation in the agent-based models. Lemarié (2003) underlines the contribution of optimization techniques to solve problems in conjunction with agent-based techniques. She proposes to use the optimisation-based models of Bader (2001) as a post-processing of an agent process, to compute final continuous transformation of a road network. In this process too, discrete transformations are applied first (to compute the most important changes), and then continuous transformation (especially to correct side effects of the discrete transformations). The models are not really merged, but used one after the other

Our opinion is that the generalization process should be seen as a sequence of transformations which could be either discrete, either continuous. For example, when applying a discrete change on a road section, it is often (even always) necessary to propagate the change to the network, and the surrounding objects, to preserve the network connectivity.

Several works have dealt with the integration of continuous transformations in the agent-based models. In (Legrand et al., 2005) and (Duchêne, 2004), two deformation methods are proposed in order to diffuse discrete changes computed during the agent generalization process to other objects (such as land use parcels). These methods provide quality improvements of the process, but these deformations are performed to the objects without taking into account their shape properties. They are considered as passive following objects. It would appear fairer to completely integrate these objects and their own shape constraints to the agent-based process. It would improve the results to confer malleable behaviour to these objects.

The problems we have to tackle to compute continuous transformations in the agent models concern the constraints and the level where the transformations have to be computed. In optimization-based models, the deformation of an object is the result of a balance between inner shape preservation constraints and external change constraints. The model is able to determine the state of the object to have such a balance. Constraints are considered as “elastic constraints”. In the agents based models, the result searched is not a balance between

constraints. Discrete operations are performed in order to satisfy totally some constraints; in case of over constrained situation, some less important constraints are relaxed. In order to compute continuous transformations, agent-based models should be able to determine balance between elastic constraints.

An other problem to tackle in order give to the object a malleable behaviour stand in the level where the deformation must be computed: in the agent based models, constraints are carried either by individual or group of objects, called micro and meso objects as presented in (Ruas, 2000) or relations between objects (Duchêne, 2004). Deformations are the result of the objects points displacement, and occur at the inner level of the objects.

### **3. Proposition: an agent-based model for continuous transformations**

In the previous part, we have presented the issue of merging optimization and agent-based models. We present now our proposition to allow to geographic objects to become malleable in the agent models. First and foremost, we give the general principles of our model, then some elements of description. Finally, first results are presented, and further works are proposed.

#### **3.1 Principles of the model**

To give the object a malleable behaviour in an agent based model, we propose:

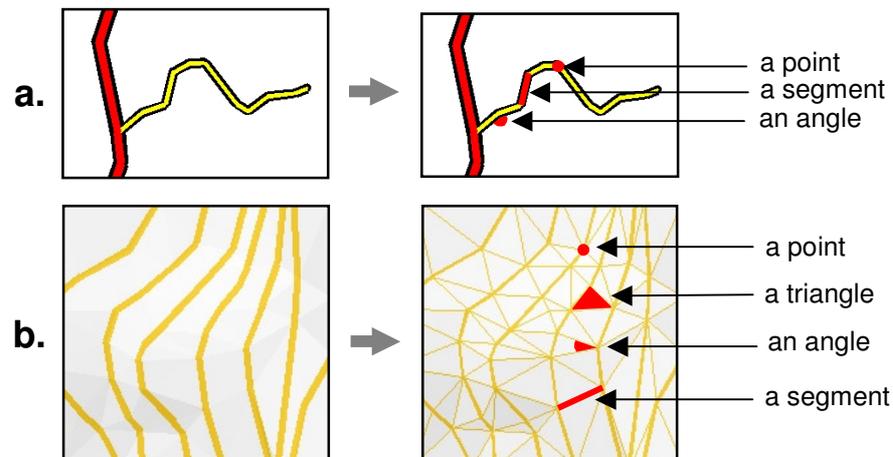
- to decompose the objects to be deformed into simple parts (points, segments, triangles, angles...). We call these parts sub-micro objects.
- to constrain these sub-micro objects (for example, the length of a segment, the distance between two points...). These constraints are elastic. Some of them compose the inner shape preservation constraints of the object.
- to compute deformations by finding a balance position between the elastic constraints. To find such a position, we propose to consider each point composing the objects geometry as an agent. The goal of each point agent is to reach a balance position between the constraints of the sub-micro objects its is belonging to.

These 3 principles, (object decomposition, elastic constraints, points Agentification) are now developed.

#### **3.2 Description of the model**

##### **3.2.1 Objects decomposition: points, segments, angles, triangles**

As presented in previous sections, a deformation is the result of points displacements in order to reach a balance position between preservation and change constraints. Some of these constraints are inner constraints, carried by parts of the object. We propose to make explicit these parts and their constraints. For example the road network (figure 2 a.) has been decomposed into points, segments and angles composing its geometry. The DTM (figure 2 b.) is represented by a triangulation, composed of triangles, segments, angles and points. These parts of objects are not geographic objects. Because it composes the micro objects, we propose to call them “sub-micro objects”.



**Figure 2.** Decomposition of a DTM and a road network into sub-micro objects.

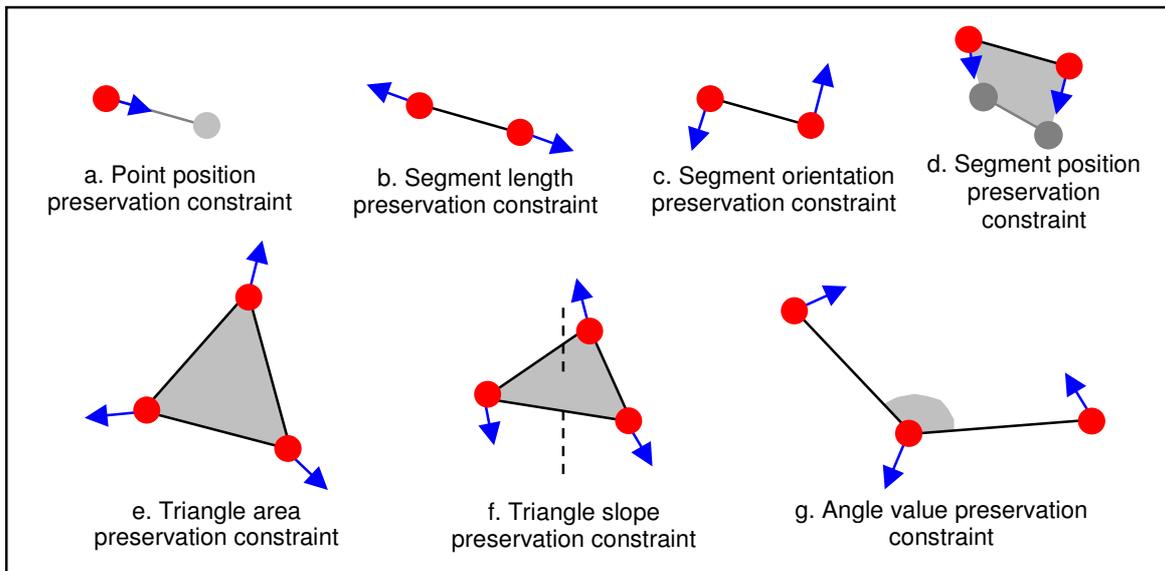
The principle of our model is to consider the points as agents, whose purpose is to reach a balance between the constraints of the sub-micro objects it belongs to. We present now some constraints we propose, and then how the point-agents act to achieve their goal.

### 3.2.2 Elastic constraints proposition

The elastic constraints we propose to make carry to the sub-micro objects are the following:

- the point position preservation constraint (figure 3 a.),
- the segment length preservation constraint (figure 3 b.),
- the segment orientation preservation constraint (figure 3 c.),
- the segment position preservation constraint (figure 3 d.),
- the triangle area preservation constraint (figure 3 e.),
- the triangle slope preservation constraint (for a DTM triangle, figure 3 f.),
- the angle value preservation constraint, (figure 3 g.).

On figure 3, we have represented these constraints: a red circles represent a point in its current state, a gray in its initial state. The blue arrows represent the influence of the constraint on the points in order to improve its satisfaction. Some of these constraints result from an adaptation of the “springs” used in (House, D. H., and Kocmoud, C. J. 1997) to perform cartograms.

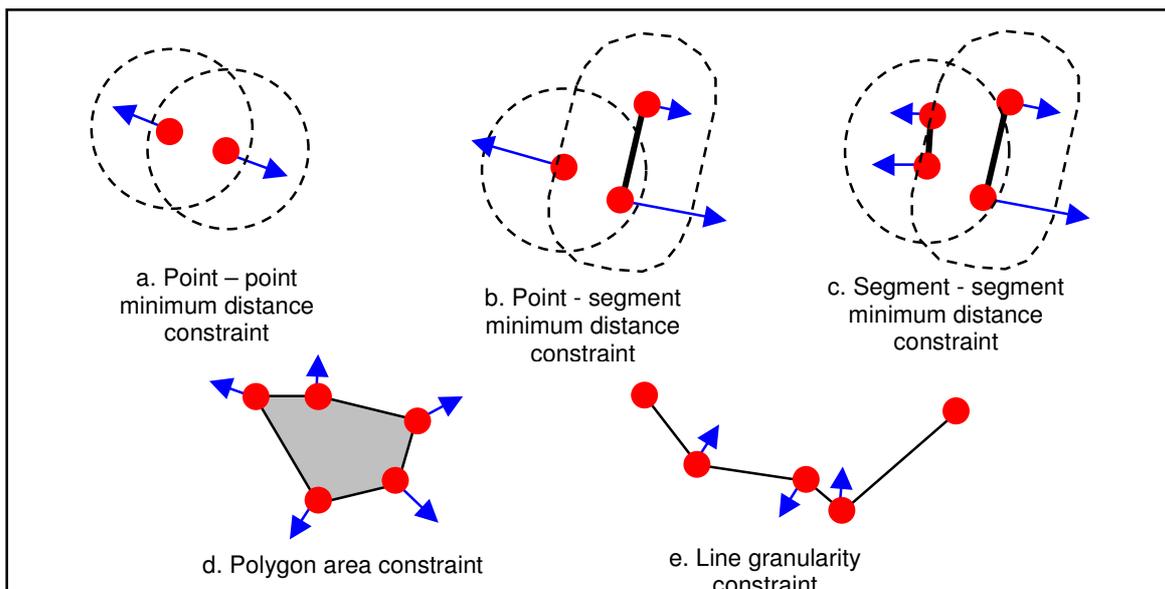


**Figure 3.** Examples of sub-objects shape constraints.

We propose to add other constraints concerning relations between sub-objects, which are not belonging to the same object:

- the minimum distance between two points constraint (figure 4 a.),
- the minimum distance between a segment and a point constraint (figure 4 b.),
- the minimum distance between two segments constraint (figure 4 c.).

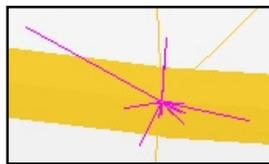
These constraints can be used to confer to the malleable objects the capability to push them. The model allows adding elastic constraints carried by objects. For example, we can define polygon area constraint (figure 4 d.) which could force a polygon to have a specific area, or a line granularity constraint (figure 4 e.), which could allow to compute an elastic smooth of the line.



**Figure 4.** Examples of sub-objects shape constraints.

### 3.2.4 Points as agents

To trigger the displacement of the points and thereby compute the deformation, points are considered as agent. Their goal is to reach a balance position between the constraints of the sub-objects it belongs to. To achieve this goal, each point is able to measure the state of its constraints. For each constraint, the point determines a displacement to compute in order to progress toward the satisfaction of this constraint. If the sum of these displacements is null, the balance position is reached (figure 5). While the sum is not null, the point compute an adequate displacement toward a global improvement of its constraints. The points progress altogether toward their own balance, until they have reached it. Further details on the way to trigger the points and to calculate the displacements for each elastic constraint are given in (Gaffuri, 2006).



**Figure 5.** A point in a balance position  
(Purple lines are displacement vectors whose sum is null).

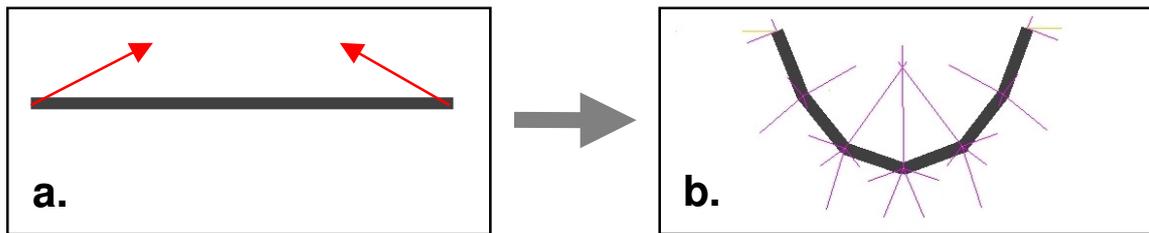
An important point of the triggering process is that only a few points are activated: because we need to activate only the points which are not in their balance position, each point has the ability to activate its neighbour. The activation of the agent propagates. The malleable objects appear like composed of “alive” points which react only in case of necessity. As a result, the deformation is a local treatment. A time consuming activation of all the points of the dataset is not needed.

The idea to compute deformations in map generalisation by considering points as agent as already been proposed in (Baeijs, 1998). The model is different and aim to be used in conjunction with the existing discrete transformation models.

## 3. Results of malleable behaviour

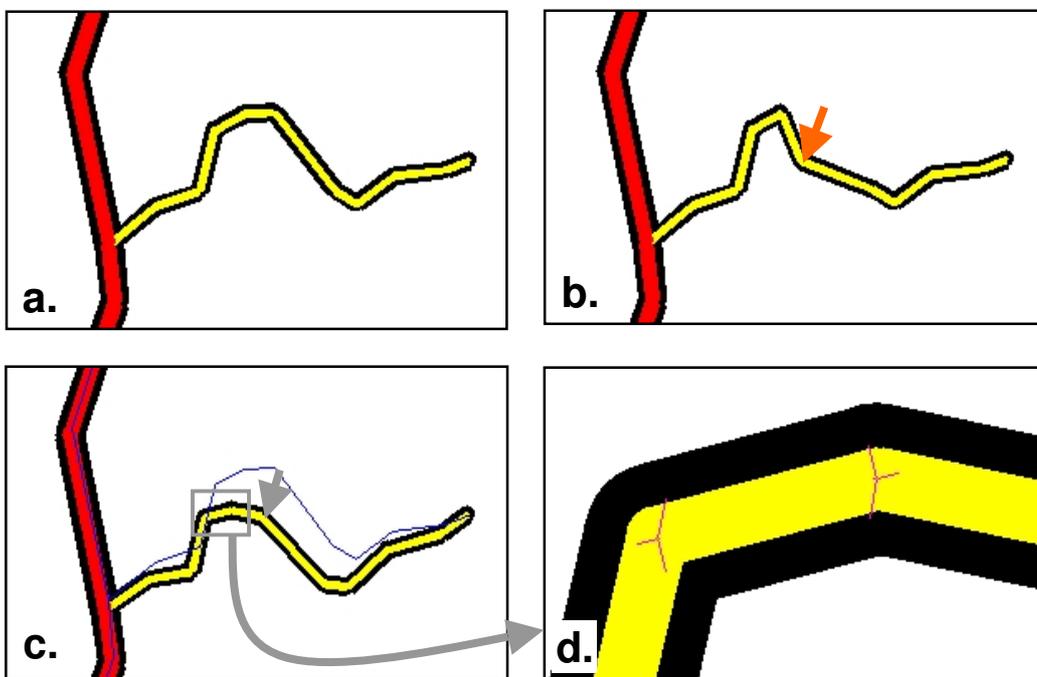
In this section, we present some results of deformations performed on several malleable objects. External displacements of some points are artificially performed (represented by the arrows). The point agents are then activated to reach their balance position: the object deforms.

Example 1: a simple line composed of 6 segments and 5 angles between them (figure 6 a.). Constraints are carried by the segments (length preservation) and angles (value preservation). 2 displacements are applied to the tip points. As a result figure 6 b., the line has bowed.



**Figure 6.** The deformation of a simple linear object.

Example 2: We consider a road network (figure 7 a.), whose a point is subjected to a displacement (b.). Constraints are carried by the segments (length preservation, and position preservation) and angles (value preservation). The displacement is diffused through the network (c.) once the points have reach a balance position (d.).

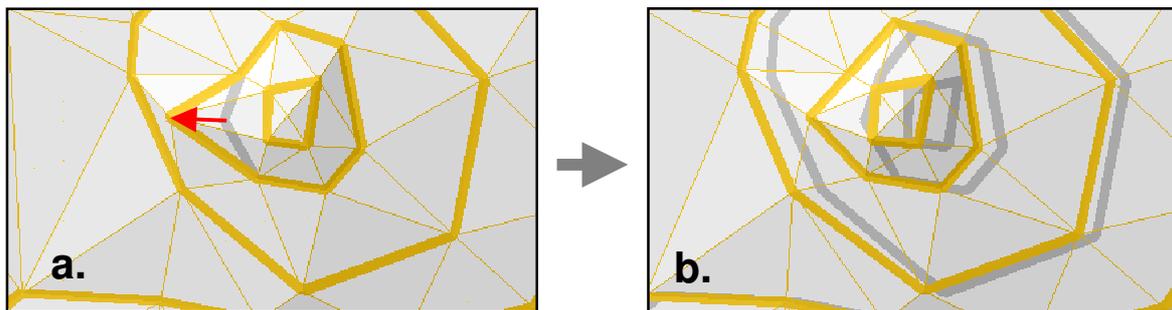


**Figure 7.** The deformation of a road network.

Example 3: The relief is represented by a Delaunay triangulation (figure 8). In this example, constraints are carried by the points (position preservation), the segments (length preservation), the angles (value preservation) and the triangles (area preservation). A displacement is applied to a point (figure 9 a.) and then propagates to its neighbours (figure 9 b.).

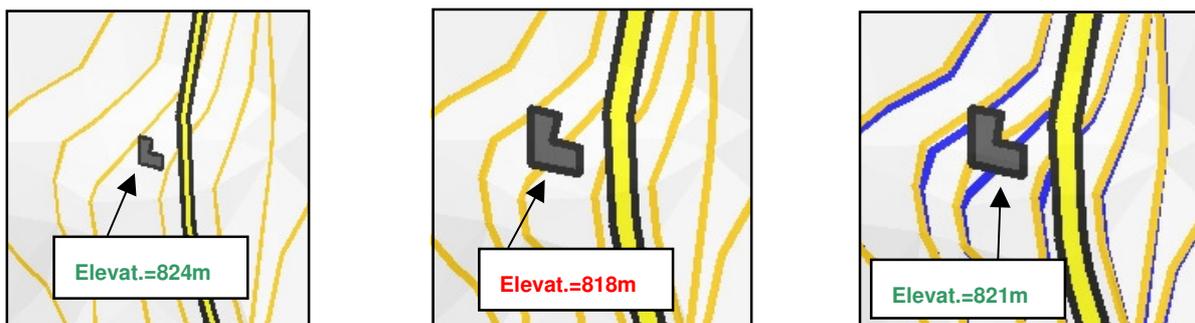


**Figure 8.** The relief field represented as a triangulation.



**Figure 9.** A malleable behaviour of the field representing the relief.

This way to deform the relief field is applied in (Gaffuri, 2005 ; Gaffuri, 2006). The purpose of this work is to allow a preservation of relations between field objects and micro objects during the generalization process. For example, the value of the elevation of a building should be preserved as much as possible. A result of this method is given figure 10.



**Figure 10.** A building deforming the relief to preserve the value of its elevation.

The presented malleable behaviours have been obtained by using some example constraints. By adding or removing elastic constraints, or tuning their relative importance values, it is possible to confer to the object some specific shape preservation capabilities. For example, we could choose to add some specific constraints to the segments composing the contour lines of the DTM (example 3). It could allow taking into account some specific shape properties of the contour lines.

### **3. Future works**

The proposed model allows computing deformations on objects. Objects have thereby the ability to have a malleable behaviour. When should an object become malleable? A future work will be to determine when a malleable behaviour should be triggered. An other issue would be to study how an object could manage these malleable behaviours: the object should be able to evaluate the result of a deformation it as been subjected to. It should have the capacity to measure if it has been too much deformed, according to some aesthetic criterions. Such a measure could be built by aggregation of the sub-micro objects constraint satisfactions. If a malleable object detects it has been too much deformed, it should have the capability to react. A possible reaction would be to give methods to the object to tune himself the importance value of its too much violated elastic constraints.

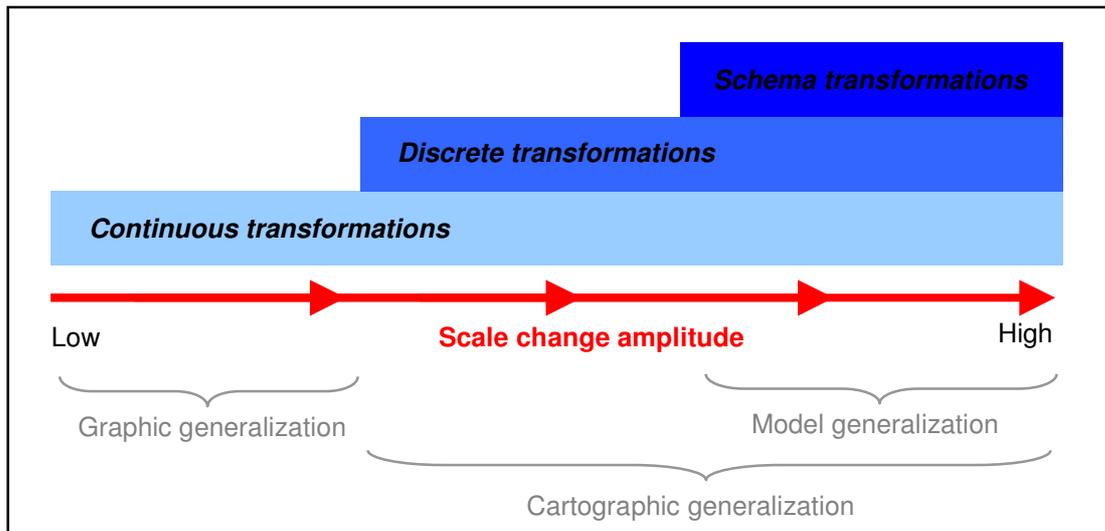
We could propose to build other elastic constraints to give new properties to the object. For example, the work of (Hunert, 2005) could be adapted to our model to allow propagating road network deformations to other objects.

### **Conclusion**

In this article, we have underlined the utility to manage both discrete and continuous transformations in a single generalization process. We have then proposed an agent-based deformation model, which allows conferring both rigid and malleable behaviours to the geographic objects. We have spread the agent-based models to manage balances between constraints carried by some parts of the objects.

This work underlines the necessity to build bridges between generalization models. Many generalization models have been developed and have allowed to progress significantly toward automation. Generalization models are applied to different application cases and are more or less adapted to solve some kinds of problems. For example, the optimization-based models are adapted to continuous transformations and agent-based models to discrete transformations as we have presented. A merging of these models in a single model gives a way to take the advantages of each of them. The interoperability between the generalization systems is not only a simple problem of programming. Some efforts in the conception of the models are required too.

The schema presented figure 11 shows the type of transformations to perform depending on the scale change amplitude: the higher the scale change amplitude is, the bigger the transformations to apply to the data are. For low scale change, only continuous transformations are sufficient. When the scale change is higher, some discrete transformations become required. For the biggest scale changes, transformations of the dataset schema are required in addition. The position of graphic, model and cartographic generalization presented in (Weibel, and Dutton, 1999) can be located on this schema. This schema illustrates the necessity for higher scale changes to use several kinds of transformations together, and especially schema transformations.



**Figure 11.** Transformations of the data functions of the scale change amplitude.

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