City Model Generalization Quality Assessment using Nested Structure of Earth Mover's Distance

Bo Mao[*,1], Hongchao Fan[2], Lars Harrie[3], Yifang Ban[1], Liqiu Meng[2],

[1] Geoinformatics, KTH, Drottning Kristinas vag 30, SE-100 44 Stockholm, Sweden
{bo.mao, yifang.ban}@abe.kth.se
[2] Department of Cartography, Technische Universität München, Arcisstr. 21, D-80333, Munich, Germany, {fan, meng}@bv.tum.de
[3] GIS-centre, Lund University, Solvegatan 12, SE-223 62 Lund, Sweden
Lars. Harrie@nateko.lu.se

Abstract. To evaluate the quality of city model generalization, an *attributed relational graph* (ARG) is used to represent the features of city models and Nested structure of Earth Mover's Distance (NEMD) is employed to calculate the visual similarity of the ARGs. The experiments show that the proposed method is coherence with user survey result.

Keywords: ARG, NEMD, generalization, similarity evaluation.

1 Introduction

Quality assessment in map generalization has been studied by the cartographic society for a long time (Shea and McMaster 1989, Mackaness and Ruas 2007, Zhang et al. 2008 2009, and Filippovska et al. 2009). The existing methods can be divided into visual, functional and quantitative assessment (Harrie 2001). In this paper we present a method for quantitative assessment. The study in this paper is on 3D city models (3DCM), but it could also be applied to 2D maps.

The method for quality assessment presented in this paper is based on *attributed relational graph* (ARG) theory (Sanfeliu and Fu 1983). Mathematically, features of city models can be represented as an ARG where the nodes represent objects like buildings or parts of buildings, and edges represent relations. Then the aspect of generalization quality assessment can be converted to the matching of two ARGs which represent the original 3DCMs and the generalized one respectively. A similar approach is presented in Fan et al. (2010). In their study they show that the ARG approach can be useful for window typification. In this work we develop the theory further and show that it also can be used for building typification.

[*] Corresponding author: Bo Mao; telephone 46 8 790-7676; fax 46 8 790-8580.

An ARG G is defined as $G = \{V, R\}$ in which $V=\{v_i|1\leqslant i\leqslant n\}$ and $R=\{r_{ij}|1\leqslant i\leqslant n, 1\leqslant j\leqslant n\}$. In our application each node in V, denoted by v_i, represents a building and some attributes such as area and height. R is $n \times n$ matrix where each element r_{ij} represents the relationship between the buildings v_i and v_j such as the distance and relative position.

A great deal of effort has been devoted to develop efficient algorithms for comparing ARGs. Kim et al. (2004) propose one such method: *Nested structure of Earth Mover's Distance* (NEMD); a method that can be used for calculating the difference between ARGs. By applying this method the similarity between an original and a generalized 3D city model can be calculated.

The remainder of this paper is organized as follows. Section 2 introduces the generation of ARG from city models. Section 3 describes the NEMD calculation with an example from Kim et al. 2004. Section 4 gives the results of comparison of NEMD and a user survey.

2 ARG generation

In order to generate an attribute related graph (ARG) G from a city model we have to compute the nodes V (representing buildings) and relationships R (representing relationship between buildings) for the graph.

Generating nodes

Each building is represented as a node where its height and ground plan are stored as attributes. In this paper, the height of building is not used for visual similarity calculation since the main view point is set as top down visualization. When computing the similarity of 3D city models, the height is important especially in street view mode.

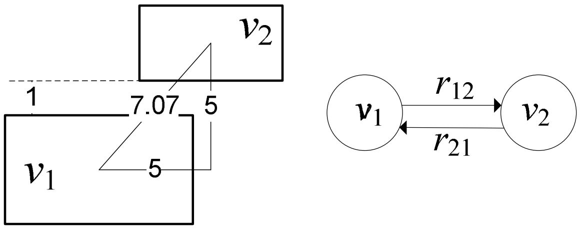
Generating relationships between the nodes

Spatial relationship is used to represent the relationship between nodes. Two common used spatial relationships, directional and distance relationship are combined to represent the relation between city objects/buildings.

For nodes v_1 and v_2 with centroid of the ground plan polygons (c_{x1},c_{y1}) and (c_{x2},c_{y2}) respectively, the directional relation from v_1 to v_2 equals to (d_x, d_y) in which $d_x = c_{x1}-c_{x2}$ and $d_y = c_{y1}-c_{y2}$. Generally speaking, direction relation from v_1 to v_2 is different from v_2 to v_1, because if v_2 is the certain direction of v_1 then v_1 should be in the opposite direction of v_2. Assume d_p is the Euclidean distance between the two ground plan polygons. The relation between v_1 and v_2, $r(v_1, v_2)=(x, y)$ in which $x=d_p *d_x/(d_x^2+d_y^2)^{0.5}$, $y=d_p *d_y/(d_x^2+d_y^2)^{0.5}$. In another words, $r(v_1, v_2)$ is the vector from centroids of v_1 to v_2 with length of polygon distance from v_1 to v_2.

Example of an ARG

Fig. 1 shows a set of buildings to the left and an ARG that represents these buildings to the right. The ARG is computed in the following way: the nodes v_1={ground plan v_1}, v_2={ground plan v_2} To calculate r_{12} the relationship between v_1 and v_2, we first get the difference value of their centroids that is (-5,-5), then scale it to their distance between the two ground plan polygons by multiplied 1/7.07. Therefore, r_{12}=(-0.707,-0.707) and similarity, the relationship from v_2 to v_1, r_{21}=(0.707,0.707).



(a) group of buildings (b) ARG

Fig. 1: Building group and its ARG

3 ARG comparison

In this section we show a methodology to compare two ARGs. Firstly we start with some general idea in the comparison and then we describe the NEMD methodology. In order to calculate the NEMD between 2 ARGs, we have to define the distance between nodes and relationship.

Distance between nodes. To compare two nodes area difference is used to represent the *distance* between polygons. The term *distance* does not here reflect the Euclidian distance between the buildings but rather their shape similarities. To compute these similarities we use the approach proposed by Filippovska et al. (2009). Based on that, the absolute area distance between P_1 and P_2 is given in formula (1)

$$D_{abs}(P_1, P_2) = 1 - \frac{Area(P_1 \cap P_2)}{Max(Area(P_1), Area(P_2))} \quad (1)$$

whereby P_1 and P_2 are moved to the same reference system with (0,0) as lower left point of their Minimum Bounding Rectangles (MBR) shown in Fig. 2(c). $Area(P_1 \cap P_2)$ is the common area of P_1 and P_2.

However, it is not enough when only the absolute area difference is considered, especially in generalization situations with zoom-in or zoom-out operations. Therefore relative area difference is introduced in this paper. Relative area difference $D_{rel}(P_1,P_2)$ can be calculated by first zooming P_1 and *P2* to make their MBRs in same width; then calculate the distance of their area difference using formula (1) as shown in Fig. 2(d). The overall distance $d_{node}(v_1, v_2)$ between two nodes of ARG is the weighted sum of absolute area difference and the relative area difference. The weight is set to 0.5 in general and could be changed according to the application requirement. Relative and overall distance is normalized in to [0, 1] in which 0 represents the complete same and 1 represents complete different.

$$d_{node}(v_1, v_2) = 0.5*D_{abs}(P_1, P_2)+0.5*D_{rel}(P_1, P_2) \quad (2)$$

For example, two rectangle P_1, P_2 in Fig. 2(a, b) have the width and height 2, 1 and 1,2 respectively. Then, the $D_{abs}(P_1, P_2)=1-1*1/Max(2,2)=0.5$, as shown in Fig. 2(c) the $Area(P_1 \cap P_2)=1*1$ and $Area(P_1)=2$, $Area(P_2)=2$. The $D_{rel}(P_1, P_2)=1-2*1/Max(8,2)=0.75$, as shown in Fig. 2(d) the P_2 is extend to the same width with P_1. The $d_{node}(v_1, v_2)=0.5*0.5+0.5*0.75=0.625$

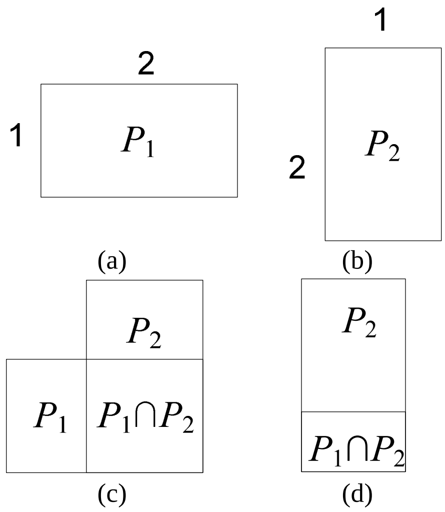


Fig. 2. An example of distance between ground plans

Distance between relations. Since the relation between city objects is vector, the Euclidean distance between vectors is used. Let $r_1 = (x_1,y_1)$ and $r_2 = (x_2,y_2)$, the normalized difference between them is $d_{relation}(r_1, r_2)= ((x_1-x_2)^2+(y_1-y_2)^2)^{0.5}/((x_1^2+y_1^2)^{0.5}+(x_2^2+y_2^2)^{0.5})$.

For example, two relations are given in Fig. 3(a) and (b) respectively. Then, in Fig.3(a), d_p=1, x=1*5/7.07 and y=1*5/7.07.Therefore $r_1=r(v_2, v_1)$=(0.707, 0.707). Similarly, $r_2=r(v_3, v_4)$=(1*4/5, 1*3/5)=(0.8, 0.6). $d_{relation}(r_1, r_2)=((0.707-0.8)^2+(0.707-0.6)^2)/(1+1)$ =0.01

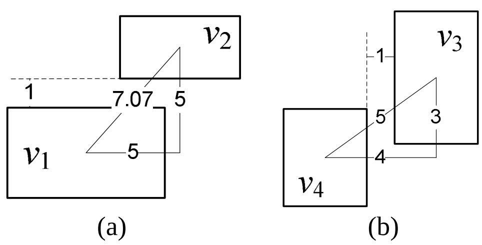


Fig. 3. An example of distance between relations

NEMD calculation

After that the distances for nodes and relationship are defined, the ARGs are matched in two steps. Firstly, the inner EMDs between every two nodes (buildings) are calculated from different ARGs respectively. Secondly, the similarity of city models is computed by using outer EMD between two ARGs. Because EMD method is used in both steps, the algorithm is called nested EMD or NEMD.

Assume that two ARGs are G and G' in which,

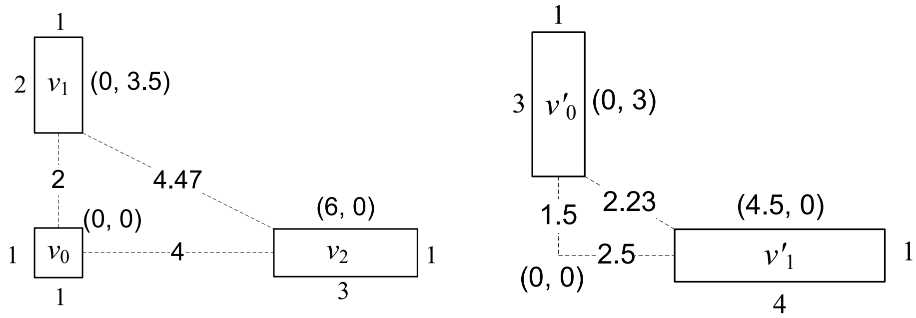
$G=\{V, R\}$, where $V=\{v_i \mid 1 \leqslant i \leqslant n\}$, $R=\{r_{ij} \mid 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant n\}$ and

$G'=\{V', R'\}$, where $V'=\{v_i' \mid 1 \leqslant i \leqslant n'\}$, $R'=\{r_{ij}' \mid 1 \leqslant i \leqslant n', 1 \leqslant j \leqslant n'\}$

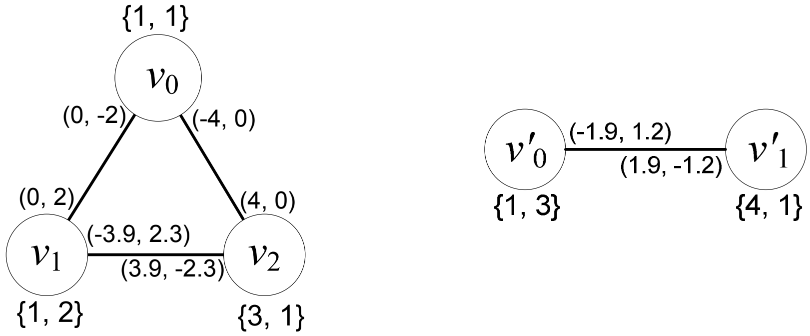
Based on the feature and relation distance, the inner EMD between two nodes v_i and $v'_{i'}$ in G and G' can be calculated as follows. First, get the inner distance matrix of v_i and $v'_{i'}$, D_{inner}. The element of D_{inner}, d_{inner} (j, j') is computed by the formula (3):

$$d_{inner}(v_i, v'_{i'}, v_j, v'_{j'}) = (1-p) \times d_{\text{node}}(v_j, v'_{j'}) + p \times d_{\text{relation}}(r_{ij}, r'_{i'j'}) \quad (3)$$

In (3), p is set to 0.5 in this paper since the node and relationship are considered as same important. This inner EMD yields one element of a distance matrix for the outer EMD, based on which NEMD of ARGs is calculated. Fig. 4 gives an example of the NEMD calculation between a building group and its generalized models.



(a) Original building group (b) Generalized building group



(c) G: Original ARG (d) G': Generalized ARG

Fig. 4. An example of ARG matching.

Fig. 4(a) shows a ground plan map of a building group with three buildings (v_0, v_1, v_2), while Fig. 4(b) is the generalized model of (a) by removing v_0 and adjusting the remaining buildings. All buildings are represented as rectangles in order to simplify the calculation. Fig. 4(a) and (b) show the distance between the buildings, centroids of the building ground plans and their width and height. Take v_2 in Fig. 4(a) for example, the distances from v_2 to v_1 and v_0 are 4.47 and 4; centroid of v_2 is (6, 0); width and height of v_2 is 3 and 1.

The corresponding ARGs are given in Fig. 4(c) and (d) respectively, in which numbers in curly braces, like {1,2}, are width and height of its nearby node and numbers in parenthesis, like (0,2), are the relationship between nodes of the edge. Take v_2 in Fig. 4(c) for example, {3, 1} means the width and height of v_2 is 3 and 1; (4,0) means the relationship from v_2 to v_0 is (4,0) as defined in section 2.

To get the NEMD value of *G* and *G'*, first we need to create the inner distance matrix D_{inner} of every node pair. For two ARGs in Fig. 4, six D_{inner} matrix will be created based on Formula (2). For example, D_{inner} of v_0 and v'_0 are given in Formula (3), in which the first element is calculated as follows:

$d_{\text{note}}(v_0,v'_0)= 0.5*D_{abs}(P_0, P'_0)+0.5*D_{rel}(P_0, P'_0)=0.5*(1-1/3)+0.5*(1-1/3)=0.667$
$d_{\text{relation}}(r_{00}, r'_{00})=0$ since $r_{00}=r'_{00}=(0,0)$
$d_{\text{inner}}(v_0, v'_0, v_0, v'_0) =(1-p)*d_{\text{note}}(v_0,v'_0)+p*d_{\text{relation}}(r_{00}, r'_{00})=0.5*0.667+0.5*0=0.333$

The second element is:

$d_{\text{note}}(v_0,v'_1)= 0.5*D_{abs}(P_0, P'_1)+0.5*D_{rel}(P_0, P'_1)=0.5*(3/4)+0.5*(3/4)=0.75$
$d_{\text{relation}}(r_{00}, r'_{01})=1$ since $r_{00}= (0,0)$ $r'_{01}=(-1.9,1.2)$
$d_{\text{inner}}(v_0, v'_0, v_0, v'_1) =(1-p)*d_{\text{note}}(v_0,v'_1)+p*d_{\text{relation}}(r_{00}, r'_{01})=0.5*0.75+0.5*1=0.875$

All elements in D_{inner} can be generated similarly with Formula (3); for the example in Fig. 4 we then receive the numerical values in Formula 4 below.

$$D_{inner}(v_0,v'_0)=\begin{bmatrix} d_{inner}(v_0,v'_0,v_0,v'_0) & d_{inner}(v_0,v'_0,v_0,v'_1) \\ d_{inner}(v_0,v'_0,v_1,v'_0) & d_{inner}(v_0,v'_0,v_1,v'_1) \\ d_{inner}(v_0,v'_0,v_2,v'_0) & d_{inner}(v_0,v'_0,v_2,v'_1) \end{bmatrix}=\begin{bmatrix} 0.333 & 0.875 \\ 0.667 & 0.910 \\ 1.056 & 0.323 \end{bmatrix} \quad (4)$$

Then, based on the created D_{inner}, we compute the D_{outer} matrix. For example $d_{\text{outer}}(v_0, v'_0)$ is calculated as follows:

Assume the number of column is N_c and the number of row is N_r, if $N_c<N_r$, d_{outer} value is the sum of the smallest values in each column divided by N_r; otherwise d_{outer} value is the sum of the smallest values in each row divided by N_c. This is the simplified d_{outer} calculation algorithm when each node has the same weight as defined in this paper. The Formula (5) shows the calculation method, in which *Row*(D_{inner}, i) is the i-th row of D_{inner}; *Col*(D_{inner}, i) is the i-th column of D_{inner}.

$$d_{outer}(v_i, v'_{i'}) = \begin{cases} \dfrac{\sum_{n=1}^{Nc} \min(Row(D_{inner}(v_i, v'_{i'}), n))}{Nr} & if\ Nc < Nr \\ \dfrac{\sum_{m=1}^{Nr} \min(Col(D_{inner}(v_i, v'_{i'}), m))}{Nc} & if\ Nc \geq Nr \end{cases} \quad (5)$$

For example the $d_{\text{outer}}(v_0, v'_0)$ from $D_{\text{inner}}(v_0, v'_0)$ is (0.333+0.323)/3=0.219. This number will be the first number in the D_{outer} as shown in Formula (6).

$$D_{outer} = \begin{bmatrix} d_{outer}(v_0, v'_0) & d_{outer}(v_0, v'_1) \\ d_{outer}(v_1, v'_0) & d_{outer}(v_1, v'_1) \\ d_{outer}(v_2, v'_0) & d_{outer}(v_2, v'_1) \end{bmatrix} = \begin{bmatrix} 0.219 & 0.260 \\ 0.153 & 0.414 \\ 0.431 & 0.153 \end{bmatrix} \quad (6)$$

In fact, the value in D_{outer} is the distance between nodes from two ARGs. For example, in formula (6), we can say the distance from v_0 and v'_0 is $d_{\text{outer}}(v_0, v'_0)$=0.219. Therefore, the minimum value in each column or row is the mapping value from one node to another. In formula (6), since $d_{\text{outer}}(v_1, v'_0)$ is the smallest value in second row, then we can say that v_1 in ARG *G* are represented by v'_0 in *G'*.

All EMD values between node pair compose the D_{outer} that can be used to calculate the NEMD value of two ARGs. In our application, not only partial but also overall difference between ARGs should be considered because generalization is imposed for whole city models. Therefore we use the sum of the smallest element in each column (if *Nc*>*Nr*) or row (otherwise) for overall matching. This can ensure that every node in one ARG has a representation node in another ARG. Hence we obtain:

$$NEMD = \begin{cases} \sum_{i=1}^{Nc} \min(Row(D_{outer}, i)) & if\ Nc > Nr \\ \sum_{i=1}^{Nr} \min(Col(D_{outer}, i)) & if\ Nc \leq Nr \end{cases} \quad (7)$$

In formula (7), *Nc* is the number of column of Douter, *Nr* is its number of row. $Row(D_{\text{outer}}, i)$ is the i-th row of D_{outer}, $Col(D_{\text{outer}}, i)$ is the i-th column of D_{outer}.

In our example, we get the NEMD of *G* and *G'*: 0.219+0.153+0.153=0.525 according to D_{outer} in Formula (4). It is also clear that the v_0 and v_1 in *G* are merged into v'_0 in *G'*; v_2 in *G* is represented by v'_1 in *G'*. This result is reasonable for generalization in Fig. 4.

4 Case study

We test the proposed quality evaluation algorithm by user survey. The original city models are typified by removing some buildings and adjusting the remaining ones. Different generalization results are generated by different typification strategies. As shown in Fig. 5, in each row, the left and right figures are typified models, the middle one are original models.

(a) 5 buildings removed
NEMD:12.86

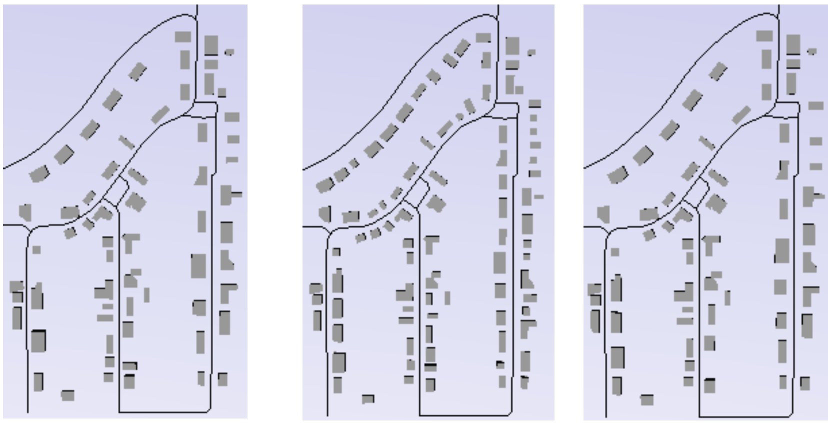
(b) Original Models

(c) 5 buildings removed
NEMD: 101.6

(d) 5 buildings removed
NEMD: 101.6

(e) Original Models

(f) 14 buildings removed
NEMD: 50.0



(g) 25 buildings removed
NEMD: 164.5

(h) Original Models

(i) 25 buildings removed
NEMD: 151.9

(j) 14 buildings removed
NEMD: 50.0

(k) Original Models

(l) 14 buildings removed
NEMD: 56.7

Fig. 5 Typified city models

Based on the above city model typification results, 10 students from KTH are asked to compare the two typification options in each row (right and left one) and decide which one is more similar with the original models (middle one). The results of user survey are given in Table 1.

Table 1. User survey results

	(a):(c)	(d):(f)	(g):(i)	(j):(l)
User survey	10:0	0:10	1:9	7:3
NEMD	12.9:101.6	101.6:50.0	164.5:151.9	50.0:56.7
NEMD difference	89.3	51.6	12.6	6.7

For the first row, all participants think that Fig. 5(a) is more similar than Fig. 5(c), which conforms with their NEMD value to the original models (Fig. 5(a) has the smaller NEMD value). Between Fig. 5(d) and Fig. 5(f), all participants select Fig. 5(f) which has smaller NEMD value than Fig. 5(d), because it is more similar to the original models. Note that only 5 buildings are removed in Fig. 5(d) while 14 are removed in Fig. 5(f). Even though, all participants consider that Fig. 5(f) is a better typification result since it preserve the overall pattern of the models, which is employed in the proposed similarity evaluation algorithm for similarity assessment.

In the first two rows, all participants can easily make their decision since the difference is quite obvious, which is indicated by the difference of their NEMD values (89.3 and 51.6 for the first and second row respectively). In third and forth rows, the typified models are quite similar from one another and the participants may give the different evaluation results. The proposed quality assessment method makes the same evaluation results with majority of the participants. We also notice that the participants are more confused as the difference between the NEMD values of the typified models. For example, in the third row, the NEMD difference between Fig. 5(g) and Fig. 5(i) is 12.6 and nine people select Fig. 5(i). Meanwhile, in the forth row, the NEMD difference between Fig. 5(j) and Fig. 5(l) is 6.7, and the number of people select Fig. 5(j) is reduced to seven. The user survey results show that different people may have different visual perception and quality assessment decision for the same typified city models. Meanwhile the proposed method can reflect the majority opinion of the participant according to the user survey.

5 Conclusions

Quality assessment algorithm is essential for automatic city model generalization. For many applications, visual similarity is the main criterion of the quality. In this paper, a method is designed to calculate the visual similarity between the generalized city model and the original one. This method first extracts the feature of each object in the city models and detects the spatial relationships between these objects. Based on these feature objects and relationships, an *attributed relational graph* (ARG) is generated to represents the city models. Then the corresponding ARG is created for its generalized models. The visual similarity between the original and generalized models is quantified by calculating the nested earth mover’s distance of their ARGs. A user survey is carried out to test our quality assessment algorithm. The results show that the proposed similarity evaluation method can reflect the visual perception of human users, and is valid to be used for generalization quality assessment.

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