Parallelity in Chorematic Territorial Outlines

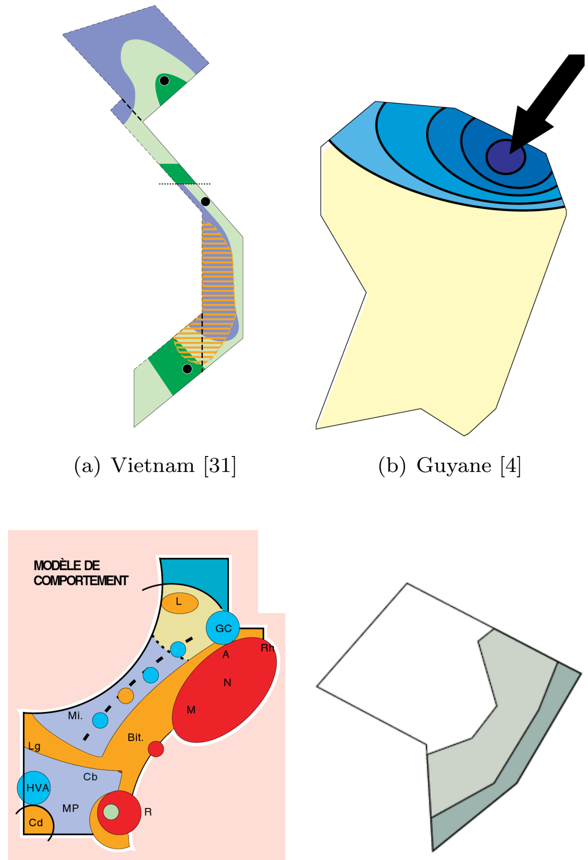
Andreas Reimer* Wouter Meulemans†

Abstract

We conjecture that parallelity is an important design rule for chorematic territorial outlines. We test this hypothesis by automatically generating outlines of high parallelity and comparing these to manually drawn chorematic outlines found in the literature. The outlines are computed by selecting characteristic points of a given territorial outline and using these as input for a simulated annealing process on the vertices and edges that attempts to maximize parallelity.

1 Introduction

Chorematic diagrams are highly abstracted depictions of complex geospatial situations. They can be used effectively to support consensus building, communication of results to a public audience, geodatabase overview or high level comparisons of spatial patterns in interactive environments [17, 24, 33]. Since the manual construction of such a diagram is also a time-consuming process, automated construction of chorematic diagrams is a worthwhile endeavor. We approach the problem via generalization. A variety of generalization operations [23] has been identified [24, 26] in an attempt to formalize salient design rules and visualization strategies. One of those operations is the chorematic schematization of territorial outlines. As can be seen in Figure 1, manually produced chorematic outlines use polygon-based, curve-based, as well as circular-arc-based approaches [25]. This paper concentrates on outlines without curve elements, in particular polygons and straight-line subdivisions. We hypothesize that one of the design rules for chorematic diagrams is to draw edges in parallel if possible. Thus we present an algorithm that computes outlines of high "parallelity".



(a) Vietnam [31] (b) Guyane [4]

(c) Languedoc-Roussillon [5] (d) Brazil [32]

Figure 1: Examples of manually drawn chorematic diagrams found in the literature.

Related work. Currently, schematization problems are mainly defined by the special case of schematic networks, e.g. metro maps. A typical (strict) interpretation is to restrict orientations to multiples of 45° (octilinear), 90° (rectilinear) or occasionally 60° (hexilinear) [9]. However, there are many examples of highly generalized, abstracted or caricatured polygonal representations that do not fit that criterium, such as those depicted in Figure 1. Even for the well-known London Tube map, the case was raised that octilinearity itself might not be a design rule, but only one way to reach schematization [27]. Supporting this case, Stott's approach to schematic map construction also allows for some relaxation of the octilinearity design rule [30].

Apart from our own findings [24, 26], published work presents only very general observations on the generalization aims specific to territorial outlines for chorematic diagrams. Published chorematic diagrams indicate that strict adherence to octilinearity is not suitable in this case. An approach that does not consider angles as horizontal geometric relations [29] has

*GFZ German Research Centre For Geosciences. areimer@gfz-potsdam.de. A. Reimer is supported by the DFG-funded METRIK research training group.

†Department of Computer Science and Mathematics, TU Eindhoven, Netherlands. w.meulemans@tue.nl. W. Meulemans is supported by the Netherlands Organisation for Scientific Research (NWO) under project no. 639.022.707.

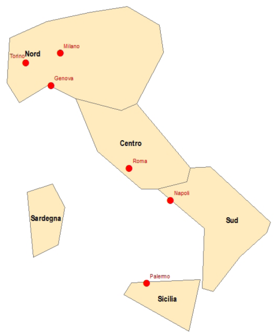


Figure 2: Automatically drawn diagram of Italy [10].

been proposed by Del Fatto [10]. Del Fatto's method does not differentiate between the schematization of the territorial outline and the thematic subdivision (i.e. area-class map [20]), and generalizes them in the same step, approaching a convex hull for every face (see Figure 2).

In a follow-up publication, Chiara *et al.* [6] do not let the outlines approach convex hulls, but equate line-simplification with producing chorematic maps. They use the following techniques for visual representation: line simplification with a topology-preserving Douglas-Peucker algorithm, on-map label placement and flow-mapping for visualizing quantitative flow information and a geographic projection (see Figure 3 and Figure 4). As no detailed information on the method is provided, we cannot compare directly. However, our own findings [24] indicate that published chorematic diagrams do not display quantitative information, very rarely use on-map labels—if at all, then as abbreviations as seen in Figure 1(c)— use very low numbers of vertices for territorial outlines and use the cartesic or conformal projections appropriate for the area of interest. Hence, their approach does not seem to follow the cartographic design rules of chorematic diagrams. Instead, they are concerned with the concept of chorematic diagrams as inspiration for interactive data exploration, making their research only orthogonally related to our work.

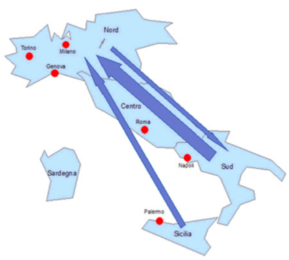


Figure 3: Automatically drawn diagram of Italy [6].

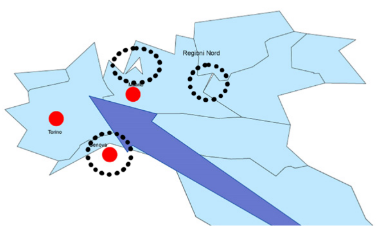


Figure 4: Detail of Northern Italy of Chiara *et al.* [6]. The position of Genova and two simplified lakes (encircled) give the impression of topology violations.

Our method to generate outlines uses the notion of characteristic points. Choosing the most characteristic points from a polyline or polygon is closely related to the wider area of (line) simplification. Line simplification has been researched widely and has several well-known algorithms including Douglas-Peucker [8] and Imai-Iri [14]. Nonetheless, several known problems exist. These problems include starting point dependency [18], parametrization to specific scales [19, 22] and the lack of a universally applicable validation (distance) measure. Chorematic diagrams depict geometries of wildly varying scale at the same level of visual complexity. For example, a city's outline in one instance is drawn with the same low number of points as that of a continent in another diagram. In generalization, algorithms are often parameterized to some target scale. In contrast, this scale has no influence in chorematic diagrams. Therefore, algorithms have to be parameterized to the desired visual style.

Hypothesis. Based on the inspection of manually drawn chorematic diagrams, we conjecture that an important design rule for chorematic outlines is *parallelity*. A high number of edges should be parallel. Preferably, parallel edges should "face" each other. By this, we mean that (part of) the orthogonal projection of one edge coincides with (part of) the other edge. This concept is illustrated in Figure 5. However, to preserve the shape of the outline, vertices are constrained to stay within a certain range of their original position. These two properties are used to develop a simulated annealing algorithm to generate outlines with high parallelity, this is presented in Section 3. This method assumes that the outline already has few vertices. Therefore, we preprocess shapes by selecting characteristic points as described in Section 2. In Section 4, we verify our hypothesis by visually inspecting generated outlines. Where possible, we compare it to the angular structure of a comparable manually drawn outline, which we consider to be the "ground truth".

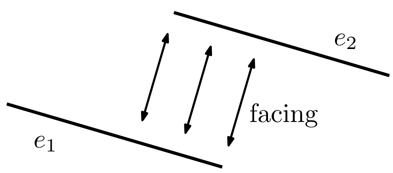


Figure 5: Edges e_1 and e_2 partially face each other.

2 Characteristic-point selection

One of the salient visual characteristics of chorematic diagrams is their minimalist, schematized design. The complexity (number of vertices) of a chorematic territorial outline typically ranges from five to fifteen. These few points bear the burden of forming a shape that is recognizable as a representation of the area of interest: they should be *characteristic points*, sometimes also referred to as critical points. For a chorematic outline, they should also be such that the outline is aesthetically pleasing. Our simulated annealing method, described in Section 3, assumes that the input has the desired complexity and that the vertices are characteristic. Hence, we require an algorithm to extract characteristic points from a polygon or subdivision.

Selection algorithm. The selection of characteristic points is closely related to line simplification. Therefore, our method builds on existing line simplification algorithms. Common algorithms such as Douglas-Peucker [8] and Imai-Iri [14] are threshold-based methods for polylines: a simplification is found such that the distance between input and output is at most the threshold ε. To find a simplification with a given number of vertices, say k, we perform a binary search to find the minimal value of ε for which the Imai-Iri algorithm produces an output with complexity at most k. Solutions with less than k points may be desirable in some cases, as adding another characteristic point actually increases the distance to the original shape. A simple example is given Figure 6.

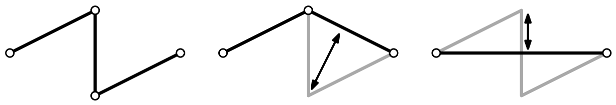


Figure 6: (a) A polyline with 4 points. (b) Simplification with 3 vertices; distance is $\frac{4}{\sqrt{11}} \approx 1.79$. (c) Simplification with 2 vertices; distance is 1.

The Imai-Iri algorithm is defined for polylines instead of polygons or subdivisions. Therefore, we must split the outline into polylines and execute the algorithm on each polyline separately. For subdivisions we cut at every vertex of degree three or higher. Now the input consists of a set of polylines and polygons. Polygons also have to be converted to a polyline by cutting at some vertex. This vertex automatically becomes a characteristic point. When a high number of vertices is used, this starting point dependency may not be much of an issue. However, since we aim for a very low complexity, the issues caused may be quite severe. An obvious solution is to try all vertices as starting point and use the best one (for example the starting point that yields the least number of vertices in the output). However, this incurs a rather large overhead, increasingly so if there are multiple polygons to be simplified simultaneously. Therefore, we use a heuristic that cuts a polygon at one of its diametrical points. This corresponds to the heuristic applied by the Douglas-Peucker algorithm which considers distant points to be characteristic for a shape.

Discussion. Strictly speaking, a requirement of the method is that the result does not intersect itself. That is, it must still be a simple polygon or subdivision after simplification. This ensures that the annealing process starts with a valid solution. The Imai-Iri method cannot guarantee that the result is free of intersections. However, due to the low target complexity, this is unlikely to occur and did not occur in our experiments. More advanced methods exist, such as the one of De Berg *et al.* [7]. This method guarantees that the result does not intersect itself. However, it cannot guarantee that a certain complexity can be achieved, as intersections are tested with the original shape of nearby polylines, rather than the simplified version.

Since the final shape is represented by few points, it is important to also consider geographic characteristic points in addition to geometric characteristic points [15]. This differentiation is often overlooked [22], but cannot be ignored in cartography. While geometric characteristic points are obtainable from the shape itself, geographic characteristic points typically require some auxiliary information. An example is the Danish-German border (Figure 7), which a viewer expects to be represented by at least two points. Purely geometric threshold-based methods consistently fail to detect this significant feature and create a triangular shape at low complexity. Using auxiliary information, our method would be able to deal with such geographic characteristic points by cutting any polyline at these points as well. However, we did not include this in our experiments.

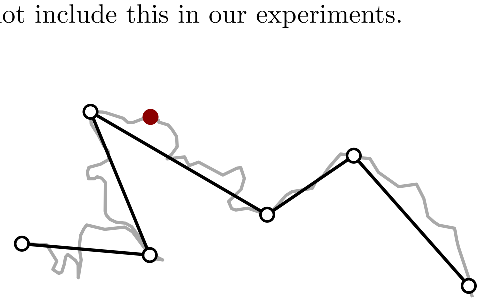


Figure 7: Northern border of Germany. The solid vertex is important to represent the Denmark-Germany border, but goes undetected by geometry-based methods when aiming at a low complexity.

3 Simulated annealing

Simulated annealing is a generic framework, often applied for optimization problems [16]. It has been used in generalization and especially schematization research [1, 2, 3]. We use the method to generate chorematic territorial outlines. Simulated annealing finds a good solution to the problem by using a heuristic local search in the solution space. Such local searches are at risk of getting stuck in local optima. The idea of simulated annealing is to have a lot of flexibility initially to escape these local optima. This flexibility is then decreased over time. Simulated annealing starts with some valid solution and tries to transform it into another solution, one which is hopefully better. A "temperature" is used to indicate and control the flexibility of the process. Based on the temperature and a random factor, it is possible to force the acceptance of a new solution, even if it is considered to be worse than the old one. If the temperature reaches zero, the annealing process stops, returning the best solution found so far. Since every vertex is not allowed to move arbitrarily far away from its original position, our solution space is guaranteed to have some maximal value, thus, after reaching temperature zero, the optimal solution so far is still modified, as long as it in fact improves the result. Algorithm 1 presents a high-level overview of the method. Simulated annealing requires two main ingredients: a quality measure for solutions, and a method to obtain a new solution from an existing solution. The quality measure Q is based on parallelity and is described in Section 3.1.

Algorithm 1 `FindChorematicOutline`($\mathcal{G}, \delta, dt$)

Require: $\mathcal{G}$ is a planar graph, δ is the threshold distance, dt is the temperature decrease

$C \leftarrow \mathcal{G}$
$O \leftarrow C$
$T \leftarrow 1$
while $T > 0$ **do**
  $r \leftarrow$ A random number between 0 and 1
  Modify C into C'
  if $\mathrm{Q}(C) < \mathrm{Q}(C')$ or $r < T$ **then**
    $C \leftarrow C'$
  end if
  if $\mathrm{Q}(O) < \mathrm{Q}(C)$ **then**
    $O \leftarrow C$
  end if
  $T \leftarrow T - dt$
end while
while a modification is made **do**
  Modify O
end while
return O

In Section 3.2, we show how to modify a solution. We present a brief discussion of our method and alternatives in Section 3.3.

3.1 Parallelity as a quality measure

The hypothesis states that parallel lines should be encouraged in chorematic diagrams. Therefore, the quality measure Q for our simulated annealing approach is based on parallelity. Every edge in the solution has its quality, $\mathrm{q}(e)$, which lies between 0 and twice its own length. The quality of a solution is then the sum over all edges, divided by twice the total perimeter length. This normalizes the score to the interval $[0; 1]$ and ensures that solutions with a longer or shorter perimeter are not preferred by default. We also wish to enforce a valid solution, one where edges do not cross and where every vertex of the solution is within a threshold distance of its original position. The threshold distance we used is 0.03 times the diameter of the shape. This corresponds approximately to what seems to be used in various manually drawn chorematic diagrams. If a solution is invalid, its quality is 0. Summarizing, the quality of a solution S is defined as follows:

$$\mathrm{Q}(S) = \begin{cases} \frac{\sum_{e \in S} \mathrm{q}(e)}{2 \cdot \sum_{e \in S} |e|} & \text{, if } S \text{ is valid} \\ 0 & \text{, if } S \text{ is invalid} \end{cases}$$

What remains is to define the quality of a single edge. As stated, $0 \leq \mathrm{q}(e) \leq 2 \cdot |e|$ must hold, for the normalization to work. The quality of a single edge consists of two parts, $\mathrm{q}_1(e)$ and $\mathrm{q}_2(e)$. The first part, $\mathrm{q}_1(e)$, is the pure parallelity score. If e is parallel to another edge, then $\mathrm{q}_1(e)$ equals $|e|$, it is zero otherwise. Adjacent edges, that share a vertex of degree two, are not taken into account. This is because we assume all the vertices to be significant: when two such adjacent edges are parallel, the shared vertex visually disappears and is no longer significant. The second part, $\mathrm{q}_2(e)$, is the "facing bonus". For every edge e' parallel to e, the overlap of e and the orthogonal projection of e' onto e is computed and added to the facing bonus, up to a maximum of $|e|$. These two parts are added to obtain $\mathrm{q}(e)$. However, this poses a problem for the simulated annealing: if an edge is not parallel to another edge, its quality is zero. Hence, the quality measure is not strong enough to distinguish between two similar solutions, where an edge is "more parallel" to another edge in one solution compared to the other. In order to steer the annealing process to better solutions, we multiply the length of the edge with the result of a Gaussian function on the minimal angle of e with any other edge, rather than giving it a binary contribution based on the existence of a parallel edge. This Gaussian function (illustrated in Figure 8) is centered at 0, has a height of 1, and a width of 0.05: $\mathrm{Gauss}(\alpha) = e^{-200 \cdot x^2}$. That is, only

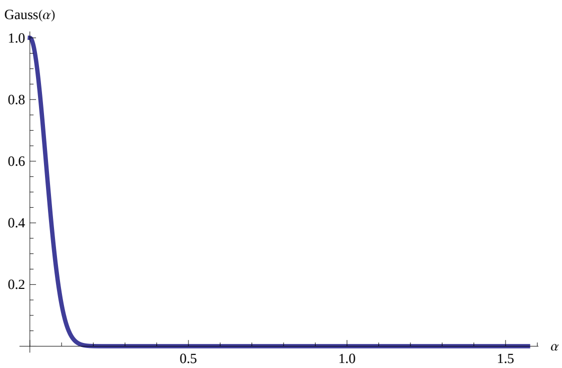


Figure 8: The Gaussian function for angles α in radians. It is positive for any α and strictly decreasing.

edges that have a small minimal angle have a significant factor. Let $\alpha(e, e')$ denote the smallest angle between two edges (with a value of ∞ for two adjacent edges) and let $\phi(e, e')$ denote the length of the overlap of the orthogonal projection of e' onto e if e and e' are parallel, it is zero otherwise. We can then summarize the above as follows:

$$\begin{aligned} \mathrm{q}(e) &= \mathrm{q}_1(e) + \mathrm{q}_2(e) \\ \mathrm{q}_1(e) &= |e| \cdot \mathrm{Gauss}\left(\min_{e' \in S \setminus \{e\}} \alpha(e, e')\right) \\ \mathrm{q}_2(e) &= \min\left(|e|, \sum_{e' \in S \setminus \{e\}} \phi(e, e')\right) \end{aligned}$$

3.2 Modifying a solution

To modify a solution, we move each vertex separately. For this we require a set of candidate moves for a vertex v. These candidate moves are generated by observing that moving v can have three effects on an adjacent edge: the orientation remains unchanged, the orientation is rotated clockwise, or the orientation is rotated counterclockwise. When the orientation remains unchanged, there are only two options left, either the edge shrinks or grows, meaning vertex v moves along the edge (or an extension of it). For each edge, these moves become candidate moves. Additional candidate moves are obtained by combining effects (e.g. rotating both edges clockwise). Every edge has two perpendicular vectors, representing a clockwise and counterclockwise rotation. The additional candidate moves are now generated by combining each such perpendicular vector of one edge with a perpendicular vector of another. Finally, not moving the vertex is also added as a candidate move. Any duplicate candidate moves are eliminated. Except for the move that keeps v in place, all candidate moves are given the same length, $\frac{1}{200}$ times the threshold distance. This results in at most $1 + 2 \cdot (|e| + |e|^2)$ candidate moves, where $|e|$ is the number of edges incident to the vertex v. For a vertex of degree two, this means nine candidate moves, combining each effect of both edges. An example of candidate moves is given in Figure 9.

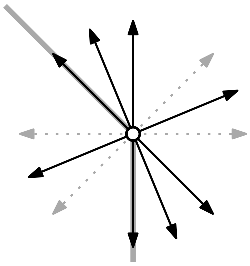


Figure 9: Candidate moves (solid black arrows) for a vertex of degree two. The dotted gray arrows indicate the perpendicular vectors used to combine effects.

For each vertex, we select a random candidate move. However, these are not applied, until a candidate move has been determined for each vertex. All these candidate moves are then applied simultaneously to obtain the new solution.

Recall that the simulated annealing process uses two variables, r and T, that express the flexibility of the process. With a small chance ($r < \frac{T}{10}$), we also allow that the entire solution is reset, to be able to escape local maxima more often. Every vertex is then set to a random position within the threshold distance of its original location.

3.3 Discussion

Another method to tackle an optimization problem is *least-squares adjustment* (LSA), a method that has been applied in generalization as well (see Sester [28]). Intuitively though, our hypothesis lends itself more for simulated annealing as we desire the reinforce good properties (parallelity), rather than weaken the bad properties: a line segment that is not parallel to any other is not necessarily a bad segment. LSA may cause edges that should be parallel to be not parallel, in order to make another edge "more parallel".

Also, there are many possible variants for simulated annealing in this context. The quality measure we propose favors long edges being parallel over short edges. With some small changes, the quality measure can be adapted such that short and long edges are weighed equally. However, we suspect that long edges are more salient than the shorter ones, the parallelity of longer edges is more important. Also, it may be desirable to incorporate into the quality measure how many edges are considered parallel to another. The effect of four edges having the same orientation is stronger than two pairs having the same orientation. This becomes mainly a concern for outlines that have a high number of (characteristic) points. In our quality measure, the problem is partially dealt with by the facing bonus. Finally, other ways of modifying a solution are possible as well. For example, one could move only a single vertex each iteration, rather than moving all simultaneously. Whether this actually improves the effectiveness of the annealing process is unclear and left as future work.

The simulated annealing process assumes that characteristic points have been selected beforehand, and that these points have been selected reasonably well. Any solution has its vertices close to their original locations. It may be possible to reduce the impact of the characteristic-point selection. Instead of requiring that each vertex stays within a threshold distance of their original location, we could for example require that the entire chorematic diagram has a Hausdorff or Fréchet distance of at most some threshold distance in comparison to the original subdivision. However, this leads to a greatly increased complexity of the algorithm. Furthermore, since the simulated annealing process requires an initialization with a valid solution, a valid solution has to be found first: for some given threshold and complexity, these are not guaranteed to exist. By decoupling the characteristic-point selection from the simulated annealing process, we guarantee the existence of a valid solution.

4 Results and discussion

In this section, we compare chorematic outlines we obtained using our algorithm to those found in the literature and discuss our findings. First, we discuss results in comparison to manually drawn outlines. After, we discuss results for subdivisions and compare it to an automatically generated outline.

4.1 Comparison to manually drawn outlines

Figures 10 to 16 show the result of our method applied to territorial outlines in comparison to manually drawn chorematic diagrams (modified to emphasize the outline). Results are shown for Argentina, Brazil, Cambodia, Guyane (twice), Spain, and Vietnam. The number of characteristic points used and the parallelity (see Section 3.1) of these chorematic outlines are given in Table 1. Note that to measure parallelity in diagrams found in the literature, a certain margin of error has been introduced. This is not only due to the accuracy of the manual work. It is a known phenomenon in perception research that angles within ensembles of other line segments and angles are systematically misperceived, appearing larger or smaller depending on context [21]. This implies that the parallelity score may be slightly lower (or higher) due to lines that appear parallel to viewer (and perhaps even the cartographer) are in fact not parallel.

We think that, for example, our results for Argentina, Brazil, Guyane, Spain and Vietnam are valid and aesthetically pleasing schematizations of their regions. To a lesser extent, this also holds for the outline produced for Cambodia. Trying to fit the method to deliver the exact same results is in danger of overfitting the process to the few examples where a ground truth is available.

Except for Argentina, the manually drawn chorematic outlines have a moderate to high parallelity measure, supporting our hypothesis that parallelity is an important design rule for territorial outlines in chorematic diagrams. We observe also that the parallelity obtained by our method is typically higher than the parallelity of the manually drawn outline. Visual inspection indicates that the manually drawn outlines are better, implying that parallelity is not the only design rule at play here. Some distortions made by our method are too significant. For example, the northern part of Vietnam (Figure 16) is compressed too much.

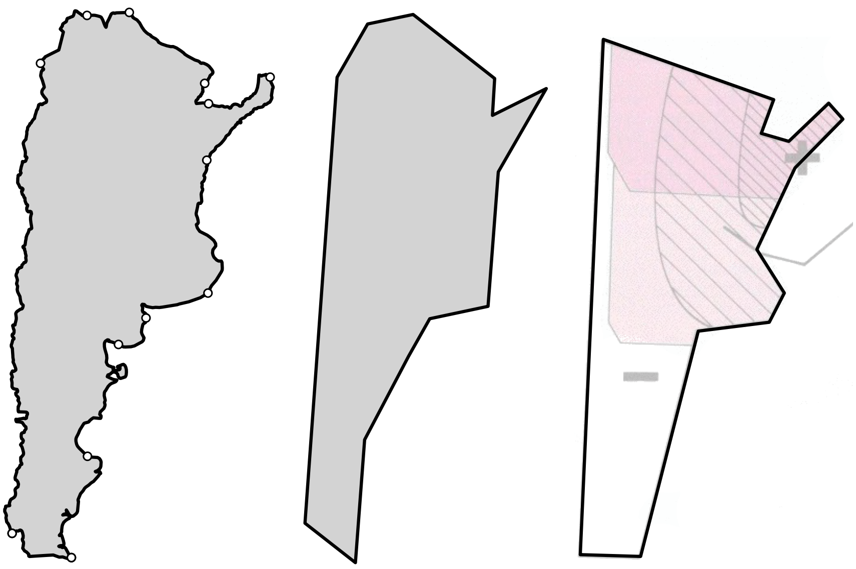
A major difference in the Guyane example (Figure 13) is the indent at the mouth of the Approuague river. It is kept in our version, but eliminated in the manually drawn version. Using a different input, we obtain a chorematic outline that corresponds more closely to the manually drawn outline (see Figure 14). The problem is a result of the method used for characteristic-point selection. Nearly all problems with characteristic points for the considered outlines can be categorized as belonging to one of three cases:

- Estuaries and large rivers
- Memorable national or regional boundaries
- Narrow territorial extrusions

Especially at the national scale, estuaries are ignored in the manual examples, if both banks belong to the region of interest depicted in the outline. On the other hand, if the estuary (or river) coincides with a boundary to a neighboring entity, the point at which boundary and river bank meet becomes an important visual anchor, as it happens with the Rio de la Plata and the Argentine-Uruguayan border. Narrow territorial extrusions, such as the Texas Panhandle, Schleswig-Holstein in northern Germany, Svay Rieng province in southeastern Cambodia or Misiones province in northwestern Argentina are cartographically important, but are consistently not detected by our point-selection method. Instead of two, only one characteristic point is selected, and a triangular shape with an acute angle is the result.

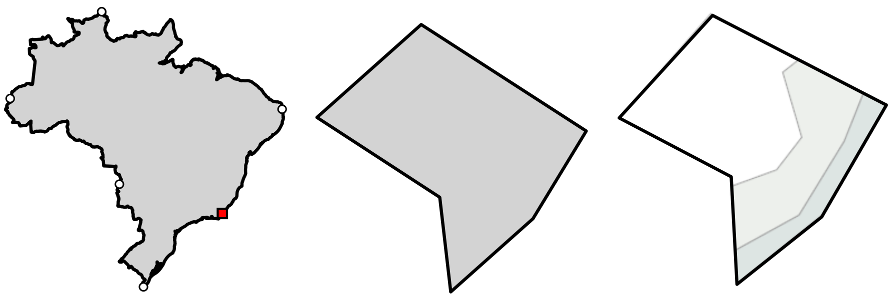
Table 1: The parallelity quality measure between our result (SA) and manually drawn outlines.

Case Territory	Points	Parallelity SA	Parallelity Manual
Argentina (Figure 10)	13	0.730	0.206
Brazil (Figure 11)	6	0.631	0.316
Cambodia (Figure 12)	9	0.542	0.307
Guyane (Figure 13)	11	0.733	0.301
Guyane (Figure 14)	11	0.726	0.301
Spain (Figure 15)	8	0.369	0.450
Vietnam (Figure 16)	12	0.780	0.762



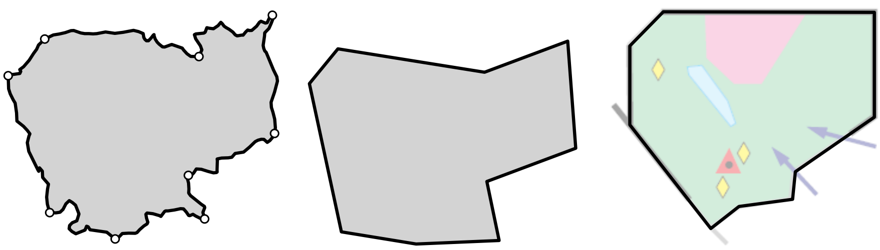
(a) Territorial outline (b) Our result (c) Manual result [13]

Figure 10: Chorematic outlines for Argentina.



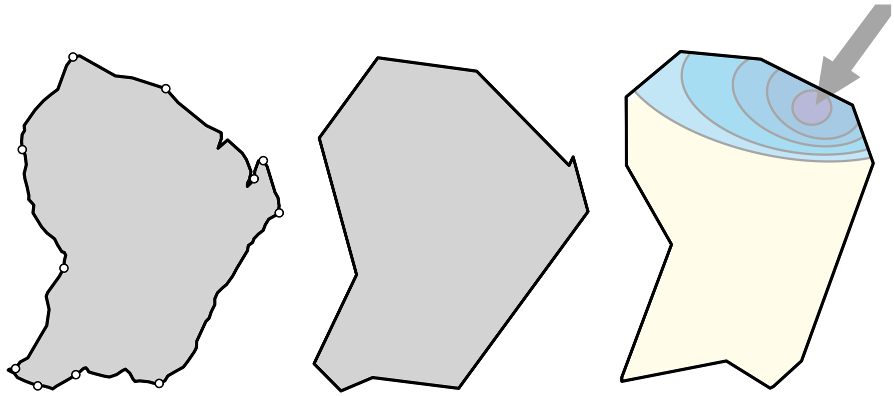
(a) Territorial outline (b) Our result (c) Manual result [32]

Figure 11: Chorematic outlines for Brazil. One geographic characteristic point (square) at Rio de Janeiro is placed manually to obtain structural correspondence with the manual result.



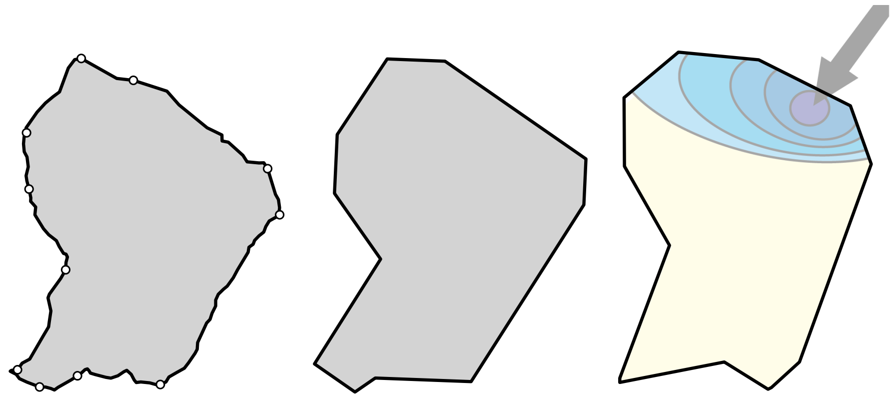
(a) Territorial outline (b) Our result (c) Manual result [12]

Figure 12: Chorematic outlines for Cambodia.



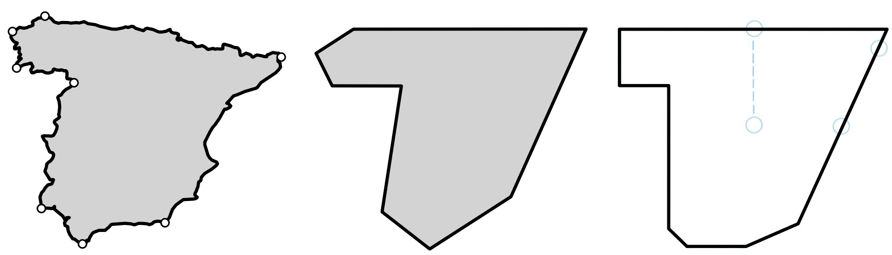
(a) Territorial outline (b) Our result (c) Manual result [4]

Figure 13: Chorematic outlines for Guyane.



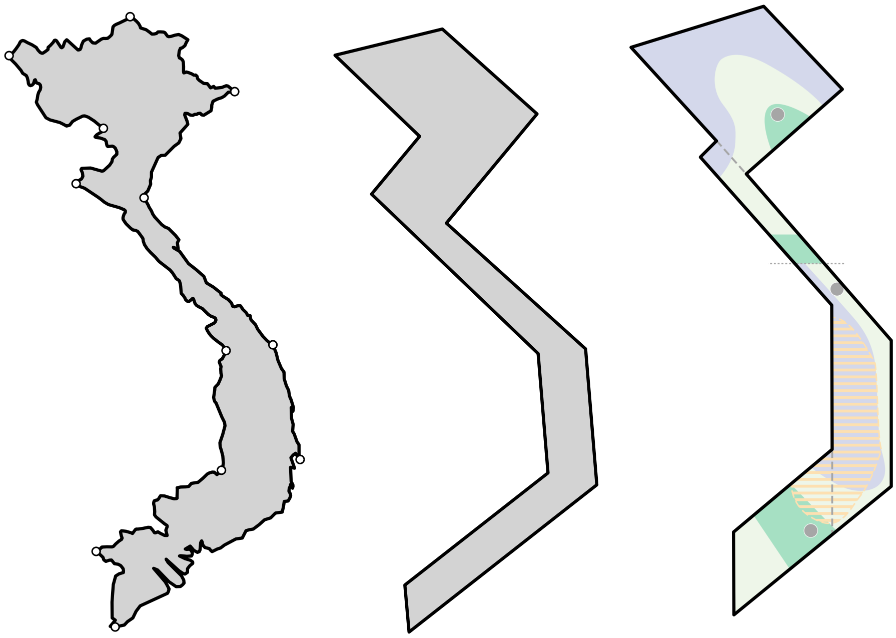
(a) Territorial outline (b) Our result (c) Manual result [4]

Figure 14: Chorematic outlines for Guyane. In the input, the Approuague estuary is merged with the mainland.



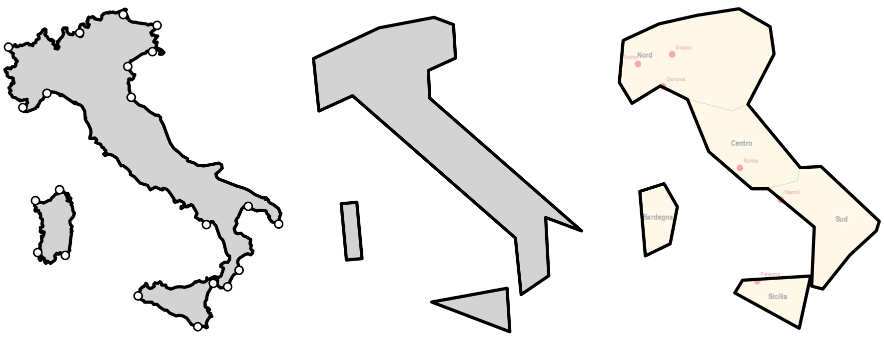
(a) Territorial outline (b) Our result (c) Manual result [11]

Figure 15: Chorematic outlines for Spain.



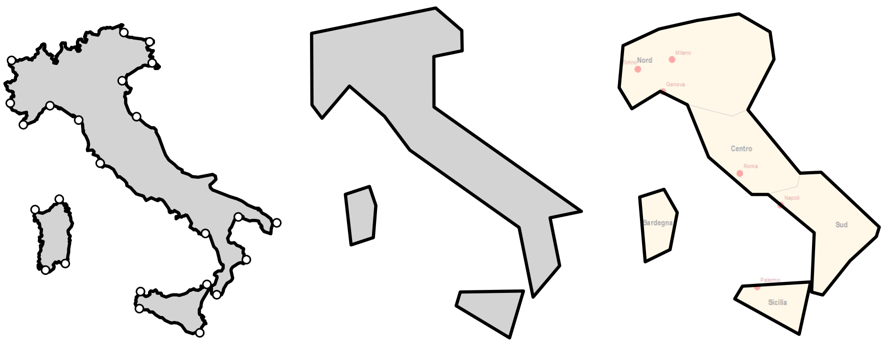
(a) Territorial outline (b) Our result (c) Manual result [31]

Figure 16: Chorematic outlines for Vietnam.



(a) Territorial outline (b) Our result (c) Automatic result [10]

Figure 17: Chorematic outlines for Italy. Our result has been obtained by treating Italy as a whole.



(a) Territorial outline (b) Our result (c) Automatic result [10]

Figure 18: Chorematic outlines for Italy. Our result has been achieved by treating islands individually.

Table 2: The angular deviation of the characteristic points (CP) and our result (SA) in comparison to the manually drawn outlines. Values are mean and standard deviation in degrees.

Territory	CP	SA
Brazil (Figure 11)	5.6° / 6.9°	4.5° / 4.6°
Vietnam (Figure 16)	7.3° / 7.4°	3.3° / 3.0°

We observe that for Brazil (after forcing one characteristic point, see Figure 11) and Vietnam (see Figure 16) the selected characteristic points in our result correspond approximately to those used by the manually drawn outline. These two cases have the same visual structure: the result shares approximate angles and facing sides with the manually drawn version. To support this claim, Table 2 shows for these two cases the average angular deviation between our result and the manually drawn chorematic outline, as well as between the selected characteristic points and the manually drawn outline. The average angular deviation is computed as $\frac{1}{N}\sum_{e \in S} \alpha(e)$, where $\alpha(e)$ indicates the smallest angle between an edge e and the corresponding edge in the manually drawn outline. This measure decreases from characteristic points to our result, indicating that the simulated annealing process moves the edges such that the used orientations are more similar to the outline found in the literature. For the other cases, at least one detected characteristic point is significantly different from the ones used in the manually drawn diagram. Therefore, this measure has no great explanatory power and thus these values have been omitted.

4.2 Results for subdivisions

Figure 17 shows our result for Italy in comparison to an chorematic outline that was automatically generated by Del Fatto [10]. Here, Italy is treated as a subdivision consisting of three polygons. Table 3 shows the parallelity measure for these outlines. It also indicates the parallelity of each of the islands in isolation. Note that, even though the parallelity of Sicily is 0, the edges of Sicily are parallel to edges of the mainland, affecting the score of the whole. The distribution of characteristic points plays an important role here. Getting a fourth point on Sicily and a fifth on Sardinia requires quite a lot of extra characteristic points on the mainland first. Therefore, we also created an outline where each island is treated separately from the mainland. This result is shown in Figure 18; Table 4 shows the parallelity scores. Note that in these results, there is no parallelity between the islands as they were treated individually.

This comparison with Del Fatto's approach [10] further hints that parallelity plays a salient role in the design of chorematic territorial outlines. The low scores

Table 3: A comparison of parallelity between our approach and Del Fatto's outlines [10] for Italy as a whole. Also see Figure 17.

Case		**Parallelity**	
Territory	**Points**	**SA**	**Del Fatto**
Italy	21/31	0.805	0.333
Mainland	14/22	0.724	0.245
Sicily	3/4	0	$\ll 0.001$
Sardinia	4/5	0.997	0.003

Table 4: A comparison of parallelity between our results and Del Fatto's outlines [10] for Italy, where islands are treated separately. Also see Figure 18.

Case		**Parallelity**	
Territory	**Points**	**SA**	**Del Fatto**
Italy	25/31	0.699	0.333
Mainland	16/22	0.801	0.245
Sicily	4/4	0.243	$\ll 0.001$
Sardinia	5/5	0.249	0.003

for Del Fatto's results prove that parallelity was not considered there; the visual comparison between both solutions suggests that our result is a better chorematic schematization for Italy. Therefore, the hypothesis of the importance of parallelity for chorematic outline schematization seems vindicated.

5 Conclusions

The results are supporting our hypothesis: while parallelity alone is not sufficient to obtain a good chorematic diagram, our results indicate that ignoring parallelity is likely to lead to unsatisfactory results. It must be noted that it is important that suitable characteristic points are selected. Hence, a study of the relation between the selection and quality of the resulting chorematic outline may be worthwhile. Also, better algorithms for selecting characteristic points can be used. Since our simulated annealing algorithm does not depend on how the characteristic points were selected, our method is easily interchanged for another.

Though it is less obvious how to define parallelity for curved segments, the use of such elements would increase the flexibility of the chorematic dramatically, allowing for a wider range of solutions. Manually drawn chorematic diagrams often combine polygonal with curved representation (such as Languedoc-Roussillon in Figure 1(c)). However, such a mixed approach hints at a design choice separate from how to represent a particular outline.

For a more complete automated generation of chorematic diagrams, we need not only a territorial outline, but also ways to generate its content, such as the curvy subdivision shown inside Vietnam in Figure 1(a). Besides generating such highly abstracted subdivision from actual data, one would need to morph the subdivision from the original outline to the chorematic outline. One could consider the points where the subdivision touches the outline as “anchor points” that can be mapped relatively straightforwardly to the chorematic outline. However, this does not account for all distortion that occurs. Ideally, one would have a continuous mapping from the interior of the original to the interior of the chorematic diagram, thereby respecting cartographic consistency.

Acknowledgements. The authors would like to thank Bettina Speckmann and Kevin Buchin for useful comments on the structure and presentation of this paper.

References

[1] M. Agrawala and C. Stolte. Rendering effective route maps: improving usability through generalization. In *Proc. of 28th Annual Conference on Computer Graphics and Interactive Techniques*, pages 241–249, 2001.

[2] S. Anand, S. Avelar, J. M. Ware, and M. Jackson. Automated schematic map production using simulated annealing and gradient descent approaches. In *Proc. of 15th Annual GIS Research UK Conference*, pages 11–13, 2007.

[3] S. Anand, J. M. Ware, and G. Taylor. Generalisation of Large-Scale Digital Geographic Datasets for Mobile GIS Applications. *Dynamic and Mobile GIS: Investigating Changes in Space and Time*, pages 161–168, 2007.

[4] S. Bourgarel. Espaces de Santé et structures spatiales en Guyane. *MappeMonde*, 94(2):46–47, 1994.

[5] R. Brunet. La population du Languedoc-Roussillon en 1990 et la Croissance Récente. *MappeMonde*, 91(1):34–36, 1991.

[6] D. Chiara, V. Del Fatto, R. Laurini, M. Sebillo, and G. Vitiello. A chorem-based approach for visually analyzing spatial data. *Journal of Visual Languages and Computing*, doi:10.1016/j.jvlc.2011.02.001, 2011.

[7] M. de Berg, M. van Kreveld, and S. Schirra. Topologically Correct Subdivision Simplification Using the Bandwidth Criterion. *Cartography and Geographic Information Science*, 25(4):243–257, 1998.

[8] D. Douglas and T. Peucker. Algorithms for the reduction of the number of points required to represent a digitized line or its caricature. *Cartographica: The International Journal for Geographic Information and Geovisualization*, 10(2):112–122, 1973.

[9] S. Dühr. *The visual language of spatial planning: exploring cartographic representations for spatial planning in Europe.* Taylor & Francis, 2007.

[10] V. D. Fatto. *Visual Summaries of Geographic Databases by Chorems.* PhD thesis, University of Salerna, Italy and INSA de Lyon, 2009.

[11] R. Ferras. *Atlas Reclus: España / Espagne / Spain.* Reclus, 1986.

[12] J.-F. Ferré. Cambodge: aléas historiques et dynamiques spatiales. *MappeMonde*, 94(1):10–14, 1994.

[13] P. Grenier. Structures et organisation de l'espace Argentin. *MappeMonde*, 88(4):36–40, 1988.

[14] H. Imai and M. Iri. Polygonal approximations of a curve – formulations and algorithms. In G. Toussaint, editor, *Computational Morphology*, pages 71–86. 1988.

[15] G. F. Jenks. Thoughts on line generalization. In *Proc. of AutoCarto 4*, pages 209–221, 1979.

[16] S. Kirkpatrick. Optimization by simulated annealing: Quantitative studies. *Journal of Statistical Physics*, 34(5):975–986, 1984.

[17] R. Laurini, F. Milleret-Raffort, and K. Lopez. A Primer of Geographic Databases Based on Chorems. In *On the Move to Meaningful Internet Systems 2006: OTM 2006 Workshops*, LNCS 4278, pages 1693–1702, 2006.

[18] Z. Li. Digital Map Generalization at the Age of Enlightenment: a Review of the First Forty Years. *The Cartographic Journal*, 44(1):80–93, 2007.

[19] Z. Li and S. Openshaw. A natural principle for the objective generalization of digital maps. *Cartography and Geographic Information Science*, 20:19–29, 1993.

[20] D. M. Mark and F. Csillag. The Nature of Boundaries on 'Area-Class' Maps. *Cartographica: The International Journal for Geographic Information and Geovisualization*, 26(1):65–77, 1989.

[21] S. Nundy, B. Lotto, D. Coppola, A. Shimpi, and D. Purves. Why are angles misperceived? *Proceedings of the National Academy of Sciences of the United States of America*, 97(10):5592–5597, 2000.

[22] P. Raposo. Piece by piece: A method of cartographic line generalization using regular hexagonal tessellation. In *AutoCarto 2010 Fall Conference Proceedings*, 2010.

[23] N. Regnauld and R. B. McMaster. A Synoptic View of Generalisation Operators. *Generalisation of Geographic Information: Cartographic Modelling and Applications*, pages 37–66, 2007.

[24] A. Reimer. Understanding Chorematic Diagrams: Towards a Taxonomy. *The Cartographic Journal*, 47:330–350, 2010.

[25] A. Reimer and D. Dransch. A process chain for the automatised construction of chorematic diagrams. To appear at ICC 2011.

[26] A. Reimer and J. Fohringer. Towards constraint formulation for chorematic schematisation tasks. In *13th ICA Workshop on Generalisation and Multiple Representation.* 2010.

[27] M. J. Roberts, E. J. Newton, and F. D. Lagatolla. Objective and subjective measures of metro map usability: investigating the benefits of breaking design rules. Manuscript, `http://tubemapcentral.com/Roberts_Metro.pdf`, 2010.

[28] M. Sester. Generalization Based On Least Squares Adjustment. In *International Archives of Photogrammetry and Remote Sensing*, pages 931–938, 2000.

[29] S. Steiniger and R. Weibel. Relations and structures in categorical maps. In *8th ICA Workshop on Generalisation and Multiple Representation*. 2005.

[30] J. Stott, P. Rodgers, J. Martínez-Ovando, and S. Walker. Automatic metro map layout using multicriteria optimization. *IEEE Transactions on Visualization and Computer Graphics*, 17:101–114, 2010.

[31] C. Taillard and V. tu Lâp. *Atlas du Viêt-Nam*. Reclus, 1994.

[32] H. Théry. *Brazil. A chorematical atlas.* Fayard/RECLUS, 1986.

[33] C. van Elzakker. *The use of maps in the exploration of geographic data*. PhD thesis, Universiteit Utrecht (Netherlands Geographical Studies 326), 2004.