

Towards general theory of raster data generalization

Paulo Raposo & Timofey Samsonov

Introduction

Despite the raster model's simplicity, deep conceptual ambiguities exist within it. Every cell covers a certain area in space, while an underlying continuous field phenomenon has variation throughout this area. A coded cell value can be determined only by using statistical techniques (e.g., mean, etc.). The same situation exists for classed rasters: cells may be crossed by several "classes", and the coded value depends on the assignment method. Also, when the cell size (i.e., resolution), shape, or orientation is changed, the resulting values for a cell at a particular location also change. Thus, we have two ambiguities: the first is *assignment ambiguity*, concerned with the possibility of different calculation schemes for raster values, and the second is the *Modifiable Areal Unit Problem* (Openshaw 1984). Raster generalization must handle both problems, and involves one or both of two processes: a) recalculation of values for new cells, and b) transition to different, likely greater, cell sizes.

Certain important cartographic and GIS applications ask for a full-fledged system of raster generalization methods. These come primarily from *thematic* and *special-purpose mapping* and analysis (e.g., surface analysis, climatological visualization and analysis, etc.). For some geographic data the raster model is more natural than vector. Among these are DEMs, geophysical fields (e.g., temperature and currents), abstract statistical fields (e.g., probability surfaces), and virtually any other continuous phenomena.

Our study aims to inventory past and current research in raster generalization both in GIScience and related fields, and to offer principles and methods as context to evolving raster generalization systems.

Background

Several conceptual frameworks for raster generalization have been proposed (Peuquet 1979, Li et al. 2001). McMaster and Monmonier (1989) divide raster generalization tasks into four categories: a) structural generalization, b) numerical generalization, c) numerical categorization, and d) categorical generalization. They identify fundamental operators and evaluate a variety of techniques used for raster-mode generalization. Comparisons of raster- and vector-based generalization have also been of interest (Daley et al. 1997, Li & Openshaw 1992, Peter and Weibel 1999). A series of articles by Li et al. on mathematical morphology in rasters should also be mentioned (Li 1994, Li & Su 1996, Su & Li 1995, Su et al. 1997a, 1997b, 1998).

Perhaps the most researched topic in raster generalization is digital elevation models (DEM) processing. Early studies in DEM generalization were based upon kernel-based filtering operations (Loon 1978). Since filtering does not take phenomenon structure into account, adaptive methods using structural information,

typically in the form of skeletal structure lines, were developed (Weibel 1987, Zaksek and Podobnikar 2005). Fan et al. (2007) developed a four-stage method using low-pass, smoothing and threshold filters that are locally combined depending on the slope and curvature of the surface. Leonowicz et al. (2009) used low and high quartile filters for valleys and watersheds, and developed a curvature-based algorithm (Leonowicz et al. 2010). Jordan (2007) developed a method based on Strahler stream order; basins are derived for streams of certain hierarchical order and these are filled by triangulating watershed boundaries; Ai and Li (2010) use a similar methodology. Other interesting fields of research are based on Fourier analysis (Clarke 1988) and wavelet transformations (Wu 2000). Jenny et al. (2011) applied wavelet multiscale pyramids to DEMs. Effects of filtering on DEM characteristics are investigated by Stauffer et al. (2012). Also, some case studies were conducted in raster land cover generalization (Monmonier 1983, Johnsson 1996, Jaakkola 1997).

Within the field of computer vision, *scale-space theory* is a formalization of multiscale image (e.g., photograph) representation, frequently used to deal with feature detection operations such as facial detection. The general approach is to convolve images with a Gaussian kernel parameterized to scale, analogous to a GIS moving-window operation. “Any coarse scale representation can be computed from any fine-scale representation using a similar transformation as the transformation from the original image” (Lindeberg 2008), meaning that multiple, sequenced representations can be computed along a continuum of scale. We propose applying the principles of scale-space theory to geospatial rasters.

Raster properties

Regular spatial sampling frequency: An intrinsic property of rasters, this allows neighborhood-scope kernel operations to work across the whole dataset without anomalous biases (projection distortions remaining invariant).

Topology & neighborhood: Every cell in a raster is clearly spatially related to its neighbors (4 or 8, depending on what kind of connectivity is used, see ‘connectivity paradox’ explained by Duff et al. 1973)—not always true for vector data. This allows intuitive analysis of every cell’s neighborhood, and thus calculation of various surface derivatives such as slope.

Projection distortions: rasters contain all the distortions of their map projection. Usually raster processing does not treat these distortions, but in case of rasters covering large extents these distortions are inevitable. A possible solution is to partition the raster, reproject each partition independently to reduce distortion (e.g., convert to UTM), process each, and finally mosaic together. Another approach may be to use a *variable kernel shape/size*, such that the kernel varies in accordance to the local projection distortion to compensate for it. This latter case is developed in our case study.

Constraints for raster generalization

We suggest that the goal is to produce generalized rasters that maintain and/or optimize their information content through successive generalizations. This can be

conceived of as a matter of avoiding or minimizing error in the generalizations. This error can be of several types, including spatial and thematic (attribute) error (Luneta et al. 1991), each of which can be quantified and statistically compared. A directly related approach is to consider the pixel entropy of successive rasters (Bjorke 1996; Knöpfli 1983; Shannon 1948).

With increasing degrees of generalization we cannot indefinitely maintain or optimize the entropy or error of a raster; at some scale and resolution the pixels will be so coarse, or the variation between pixel values so minimal, as to make this impossible. We hypothesize that that this point of failure, similar to Ratajski's (1967) *points de généralization*, is deterministically different for every input raster. If this is true, identifying that point would be very useful, since it would describe the range of scales over which a certain image is analytically reliable.

Some further possible constraints for raster generalization:

1. Geometry constraints (shape, area)
 - Horizontal boundary shapes should be preserved continuously
 - Areas and value distributions should be preserved continuously
2. Value constraints (statistics)
 - General statistics (mean, range, etc.) should be preserved continuously

In addition to novel innovative pixel operations, generalization processes may take the form of sequences of established raster processes (e.g., filters, pan-sharpening, opening and closing ["expand" and "shrink"]).

Strategies of generalization

Classed rasters have inherent "shapes" coded into them, and generalization of these may borrow techniques from vector generalization by operating on their boundaries. Continuous rasters require different approaches, probably involving calculus techniques on surface derivatives (e.g., Tobler 1979), as well as pattern-recognition methods that detect structures in the surface using edge detection algorithms and measures of spatial autocorrelation in pixel values.

Unexplored topics

McMaster & Monmonier (1989) offer a framework of four classes into which raster generalization procedures may fall. While many elementary, pixel-level operators are well-documented among varying disciplines such as GIScience, computer graphics, and medical imaging, there remain unexplored topics relevant to cartographers and spatial analysts. The issue of map projection distortion, in particular, is important whenever the scale of a map is small enough that Earth's curvature cannot be ignored. Such an issue would be encountered by an analysis seeking to identify large-scale phenomena (e.g., a hurricane) from small-scale phenomena (e.g., wind speed recorded in 1 meter pixels). In this paper we examine the consequences of ignoring the variable latitudinal scaling in the Mercator projection across a large north-south extent in raster low-pass filtering; we offer a solution using variable-size kernels parameterized to areal distortion.

Further topics to address include the information content of raster images, and the evaluation of generalized rasters with regard to differences in information, or, equivalently, error. The Modifiable Areal Unit Problem remains an important, but largely unresolved issue in spatial analysis and representation since the work of Openshaw (1984); while not limited pixels, it is centrally important in any kind of structural raster generalization. Procedural issues yet to be explored include the effects of varied sequences of operators applied to rasters, and the effects of star vs. ladder approaches (Stoter 2005) with respect to particular operators.

Case study: Raster processing with variable kernel shape

Both local and global raster processing operations are significantly affected by projection distortions — a fact frequently ignored. An example consequence of this is how slopes in polar areas are heavily flattened using a DEM in the Mercator projection. In contrast, raster processing is motivated not by pixel re-projection, but by altering the properties of the modeled field or set of objects; when a cartographer performs DEM smoothing, she probably wants to smooth every terrain feature to an equal degree, and not have features at high latitudes generalized less than those at low ones.

To treat this issue we offer an approach using variable kernel shapes that are morphed at each pixel according to the local Tissot ellipse of distortion. In this case it is important that the initial kernel size should be defined not in pixels, but in raster projection units (meters).

The following algorithm is applied:

1. Define initial shape of the kernel (commonly rectangular or ellipse) and its sizes in both X and Y directions
2. Calculate the extent of the raster in geographical units (degrees).
3. Sample raster area by the control points, which are equally spaced in degrees. Sampling distance is defined by user or can be calculated as a function of raster geographical extent and resolution.
4. Calculate the parameters of distortion ellipse at each point using projection equations. These parameters are m (meridian scale), n (parallel scale) and θ (local angle between parallel and meridian).
5. Using distortion ellipse parameters, define the local matrix of affine transformation.
6. Transform initial kernel shape and rasterize it. Round the size of the kernel to the odd number if needed.
7. For each pixel in the initial raster find the closest control point and assign its number to the pixel.
8. Process the whole raster using kernels from assigned control points.

We selected a combination of mean filtering and the Mercator projection for preliminary testing of the approach. The Mercator projection's linear distortions are calculated simply as $m = n = 1/\cos(B)$ at every point, where B is the latitude, and $\theta =$

$\pi/2$. Thus the kernel need only be scaled according to value of m . Control points were sampled each degree by latitude.

The source raster was a 5 km resolution DEM covering the European part of Russia. First, we processed the DEM using a 35×35 km (7×7 pixels) square kernel in a traditional, non-variable-kernel approach. Then we applied our variable kernel approach, with size varying from 7×7 pixels at 30° to 67×67 at 84°, always covering approximately the same 35×35 km area. Results of filtering show that our approach produces more geographically sound results than simple filtering. Compare the Kola peninsula in the northern part of the map and the Caucasus in the southern part (Figure 1). Enlarged terrain features at high latitudes appear to be generalized much less than southern territories by simple filtering. In contrast, application of a variable kernel shape smooths features of the same *geographical* area, which can be important if the smoothing is performed for geographical analysis applications.

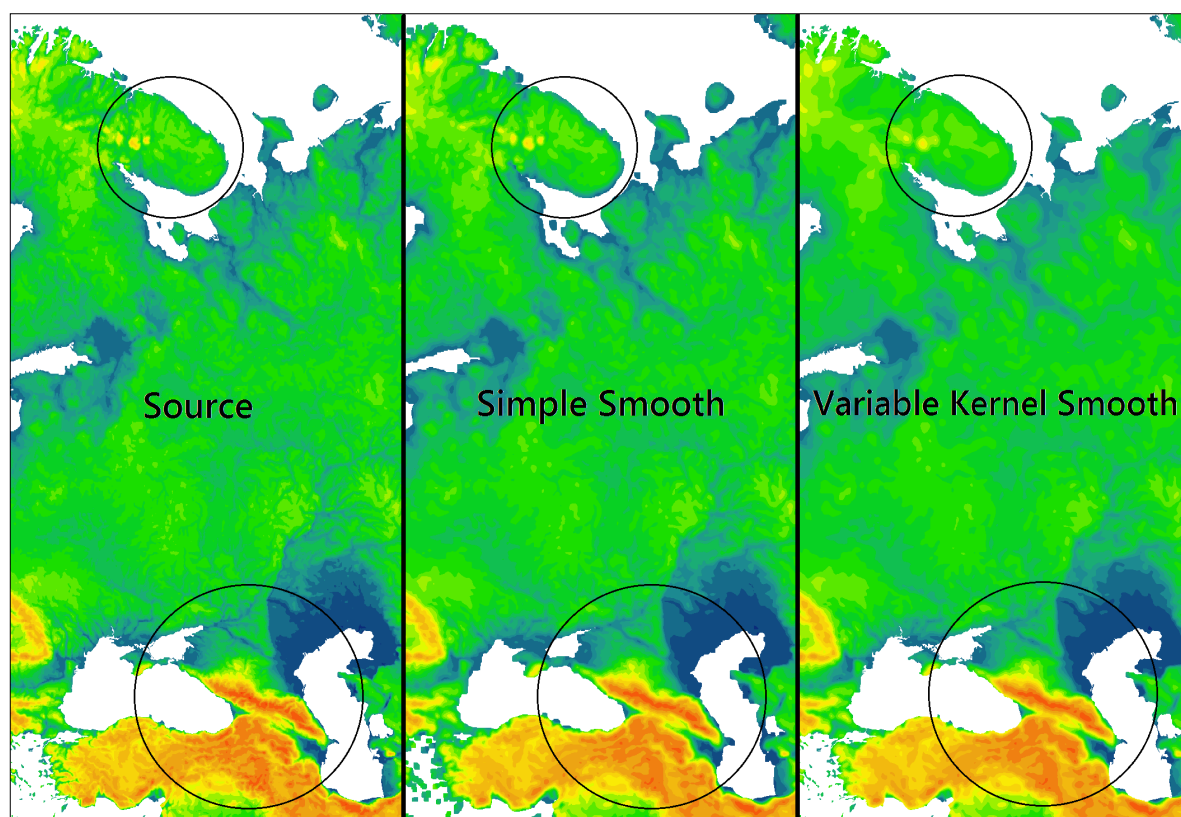


Figure 1 – Mercator projection.

The difference becomes more evident when resulting and initial rasters are re-projected into Albers Equal Area projection (50° central meridian, Figure 2). In this case it can be seen that non-variable generalization is inconsistent throughout the area, having very detailed polar regions and smoothed subtropics. And in the case of the variable kernel shape, the generalization is the same over the whole area (particularly from south to north), showing that this map introduces little projection distortions. Figure 3 illustrates the same behavior using contour lines; a greater spatial frequency and complexity of contour lines derived from the non-variable-kernel smoothing indicates that it has failed to generalize northern latitudes to the same degree as more southerly ones in terms of *generalizing the features similar in size*.

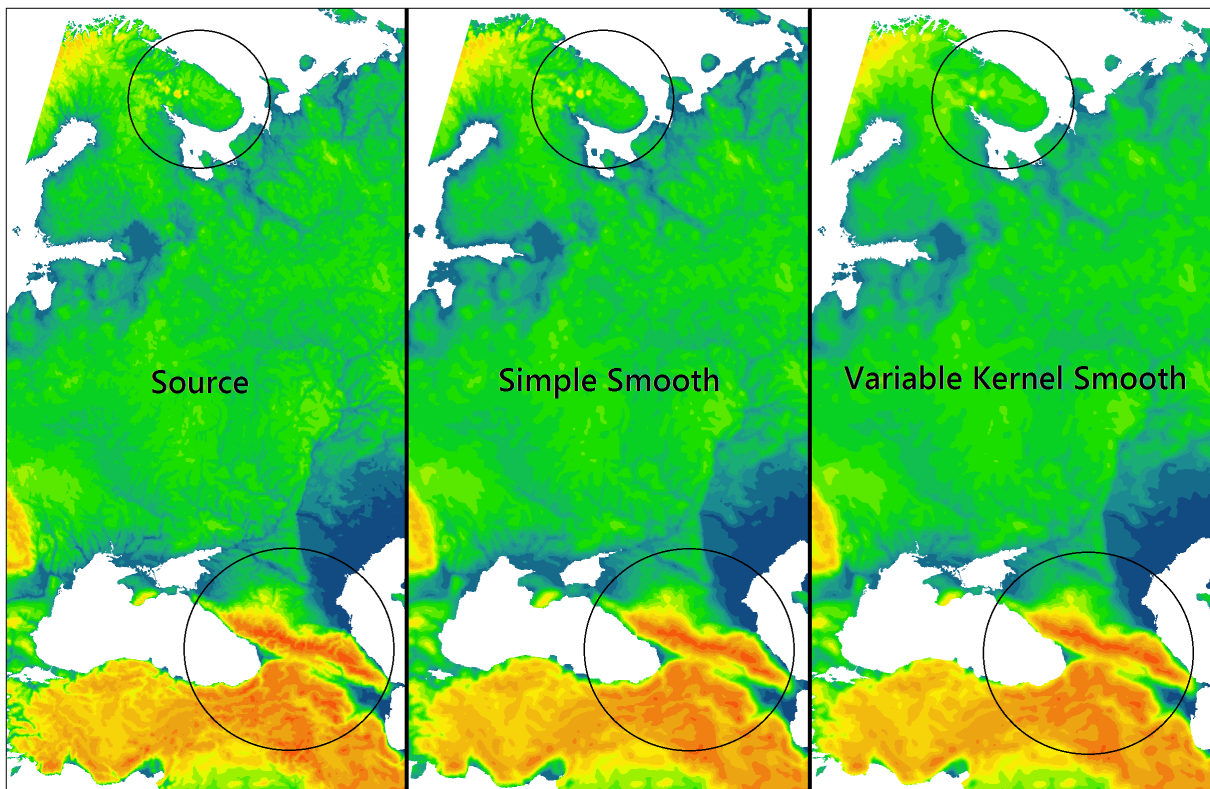


Figure 2 – Equal Area Conic projection.

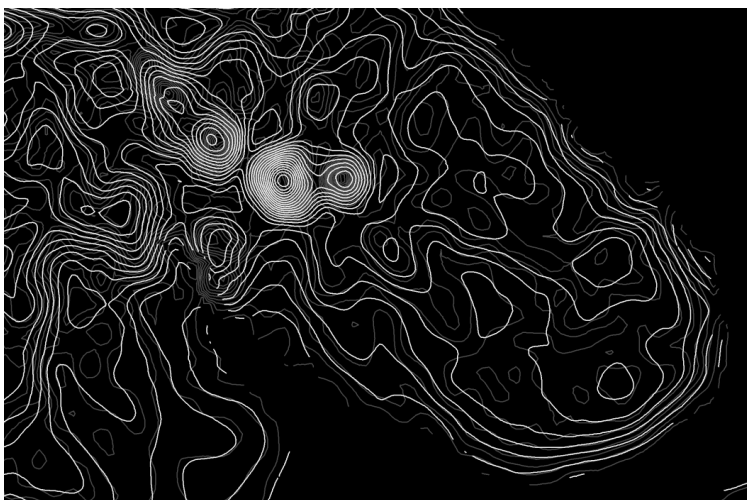


Figure 3 – 30m contours derived from non-variable-kernel DEM smoothing (grey) and variable-kernel DEM smoothing (white) at the Kola Peninsula in Russia's far north.

Distortions should also be taken into account for calculation of global raster statistics. While in the case of raster minimum and maximum all pixels should be considered, the mean and standard deviation characteristics should better be taken from a distortion-aware approach. In this case the field of areal distortion can be calculated, then normalized and used as probability density function where higher density is in the areas with smaller area distortions. Then random sampling with a given density of points can be used to extract the values and calculate mean and standard deviation of raster values.

Conclusions

We have illustrated that the generalization of rasters is a nuanced procedure that requires attention to several spatial and statistical sources of error. Most notable among these are the effects of projection distortion across large areas, which make supposedly uniform pixels in fact not uniform, and the Modifiable Areal Unit Problem, which confounds statistical aggregation and analysis, such as seen in raster coarsening and resampling. We have begun to address the issues raised by implementing a method using variably-sized kernels which are calibrated using the areal error equations of the map projection used. Further work will address statistical aggregation techniques and methods of measuring structure and information in continuous raster surfaces to enable characterization and evaluation of generalized images.

Literature Cited

- Ai T., Li J.* (2010) A DEM generalization by minor valley branch detection and grid filling. **ISPRS Journal of Photogrammetry and Remote Sensing**, Volume 65, Issue 2, March 2010, pp. 198-207
- Bjorke, J. T.* (1996). Framework for Entropy-based Map Evaluation. **Cartography and Geographic Information Systems**, 23(2), 78-95.
- Clarke K. C.* (1988) Scale-Based Simulation of Topographic Relief. **Cartography and Geographic Information Science**, 15(2), pp. 173-181(9).
- Daley N., Goodenough D.G., Bhogal A.S., Bradley Q., Yin Z.* Comparing Raster and Object Generalization. **IGARSS '97 3-8 Aug 1997. Remote Sensing - A Scientific Vision for Sustainable Development**, IEEE International 1997, pp. 677-679
- Duff, M. J. B., Watson, D. M., Fountain, T. J., & Shaw, G. K.* (1973). A Cellular Logic Array for Image Processing. **Pattern Recognition**, 5, 229-247.
- Fan Q., Yang L., Hu P.* (2007) DEM generalization based on analysis of geometry and landscape context. In: Wang, Y., Li, J., Lei, B., Yang, J. (Eds.), **MIPPR 2007**. Vol. 6790, SPIE, Bel-lingham, Washington, pp. 679035
- Guilbert E., Gaffuri J, Jenny, B.* Terrain Generalisation. In: **Abstracting Geographic Information In a Data Rich World: Methodologies and Applications of Map Generalisation**. Lecture Notes in Geoinformation and Cartography, Springer, 2014, pp. 227-258.
- Jaakkola, O.* (1998): Multi-Scale Categorical Databases with Automatic Generalization Transformations Based on Map Algebra. **Cartography and Geographic Information Systems**, 25(4):195-207.
- Jaakkola O.* Automatic Iterative Generalization For Land Cover Data. **Proceedings of AutoCarto-13**, April 7 - 10, 1997, Seattle, Washington. p. 277—286.
- Jenny, B., Jenny, H. and Hurni, L.* (2011) Terrain Generalization with Multi-scale Pyramids Constrained by Curvature. **Cartography and Geographic Information Science**, 38-1, p. 110–116.

Johnsson K. Generalization of Image Data to GIS Polygons for change detection and data base revision. **International Archives of Photogrammetry and Remote Sensing**. Vol. XXXI, Part 84. Vienna 1996, pp. 384-389

Jordan G. (2007) Adaptive smoothing of valleys in DEMs using TIN interpolation from ridge elevations: an application to morphotectonic aspect analysis. **Computers & Geosciences** 33, pp. 573–585.

Knöpfli, R. (1983). Communication Theory and Generalization. In D. R. F. Taylor (Ed.), *Graphic Communication and Design in Contemporary Cartography* (pp. 177-218). New York: John Wiley & Sons.

Leonowicz, A. M., Jenny, B. and Hurni, L. (2009) Automatic generation of hypsometric layers for small-scale maps. **Computers & Geosciences**, №35, p. 2074–2083.

Leonowicz, A.M., Jenny, B. and Hurni, L. (2010) Automated reduction of visual complexity in small-scale relief shading. **Cartographica**, 45-1, p. 64–74.

Li, Z., Mathematical morphology in digital generalization of raster map data, **Cartography**, 23(1), 1–10, 1994.

Li Z., Openshaw S. Algorithms for Automated Line Generalization Based on a Natural Principle of Objective Generalization. **International Journal of Geographic Information Systems**, 1992, 6,5, pp. 373-389.

Li, Z. L. and Su, B., Algebraic models for feature displacement in the generalisation of digital map data using morphological techniques, **Cartographica**, 32(3), 39–56, 1996.

Li Z. (2007) Algorithmic Foundation of multi-scale spatial representation. CRC Press, 310 p.

Li B., Wilkinson, G.G., Khaddaj, S. Cell-based Model For GIS Generalization. **Proceedings of the 6th International Conference on GeoComputation**. University of Queensland, Brisbane, Australia 24 - 26 September 2001, 6 p.

Lindeberg, T. (1994). Scale-space theory: A basic tool for analysing structures at different scales. **Journal of Applied Statistics**, 21(2), 224-270.

Lindeberg, T. (2008). Scale-space. In B. Wah (Ed.), **Encyclopedia of Computer Science and Engineering** (Vol. IV, pp. 2495-2504). Hoboken, New Jersey: John Wiley and Sons.

Lunetta, R. S., Congalton, R. G., Fenstermaker, L. K., Jensen, J. R., McGwire, K. C., & Tinney, L. R. (1991). Remote Sensing and Geographic Information System Data Integration: Error Sources and Research Issues. **Photogrammetric Engineering and Remote Sensing**, 57(6), 677-687.

McMaster R.B., Monmonier M. (1989). A conceptual framework for quantitative and qualitative raster-mode generalization. **GIS/LIS '89**: p. 390-403.

Monmonier, M. (1983). Raster-mode area generalization for land use and land cover maps. **Cartographica**, 20(4): 65-91.

Openshaw, S. (1984). *The Modifiable Areal Unit Problem*. Norwich, England: Geo Books, Regency House.

Pantazis D.N., Karathanasis B., Kassoli M., Koukofikis A., Stratakis P. Morphing Techniques: Towards New Methods For Raster Based Cartographic Generalization. Proceedings of the 24th International Cartographic Conference, 11 p.

Peter B., Weibel R. Using Vector and Raster-Based Techniques in Categorical Map Generalization. **Proceedings of Third ICA Workshop on Progress in Automated Map Generalization**, Ottawa, 12-14 August 1999, 14 p.

Peuquet D.J. Raster processing: an alternative approach to automated cartographic data handling, **American Cartographer**, vol. 6, 1979, pp. 129—139;

Schylberg, L. (1993). Computational Methods for Generalization of Cartographic Data in a Raster Environment. Doctoral thesis, Royal Institute of Technology, Department of Geodesy and Photogrammetry, Stockholm, Sweden.

Shannon, C. E. (1948). A Mathematical Theory of Communication. **The Bell System Technical Journal**, 27, 379-423.

Stauffer A.J., Battenfield B.P., Stanislawski L.V. Effects of Iterative Spatial Filtering on DEM Data Structure. **Proceedings — AutoCarto 2012 — Columbus**, Ohio, USA — September 16-18, 2012, 12 p.

Stoter, J. E. (2005). *Generalisation Within NMA's in the 21st Century*. Paper presented at the **International Cartographic Conference**, A Coruña, Spain.

Su, B. and Li, Z. L., An algebraic basis for digital generalisation of area-patches based on morphological techniques, **Cartographic Journal**, 32(2), 148–153, 1995.

Su, B., Li, Z. L. and Lodwick, G., Morphological transformation for the elimination of area features in digital map generalization, **Cartography**, 26(2), 23–30, 1997a.

Su, B., Li, Z. L., and Lodwick, G., Algebraic models for collapse operation in digital map generalization using morphological operators, **GeoInformatica**, 2(4), 359–382, 1998.

Su, B., Li, Z. L., Lodwick, G., and Müller, J. C., Algebraic models for the aggregation of area features based upon morphological operators, **International Journal of Geographical Information Science**, 11(3), 233–246, 1997b

Tobler, W. R. (1979). Smooth Pycnophylactic Interpolation for Geographical Regions. **Journal of the American Statistical Association**, 74(367), 519-530.

Tobler, W (1988) Resolution, Resampling, and All That. In Building Data Bases for Global Science, Mounsey, H. and Tomlinson, R. (eds.). London: Taylor and Francis, pp. 129-137.

Usery, L., Finn, M., Scheidt, D., Ruhl, S., Beard, T., and Bearden, M. (2004) Geospatial Data Resampling and Resolution Effects on Watershed Modeling: A Case Study Using the Agricultural Non-Point Source Pollution Model. **Journal of Geographical Systems**, 6, 3, pp. 289-306.

Veregin H., McMaster R. Data Quality Implications Of Raster Generalization, Proceedings of AutoCarto-13, April 7 - 10, 1997, Seattle, Washington. pp. 267-276

Wang Zh., Li Q., Yang B. (2008) Multi-Resolution Representation of Digital Elevation Models With Topographical Features Preservation. **The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences**. Vol. XXXVII. Part B6b. Beijing.

Weibel R. (1987) An Adaptive Methodology for Automated Relief Generalization. **Proceedings of Auto-Carto 8**, Baltimore, Maryland, USA, pp. 42–49.

Wu F. (2000) A Model for Multi-scale Handling and Representation of Relief Based on Wavelet Analysis. **Proceeding of the 3th surveying and mapping workshop across strait**, Hong Kong, pp. 435–441.

Zakšek K., Podobnikar T. (2005) An effective DEM generalization with basic GIS operations. **8th ICA WORKSHOP on Generalisation and Multiple Representation**. A Coruña, July 7-8th.